Uncertainty Shocks, Equity Financing, and Business Cycle Amplifications*

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Abstract

I propose a transmission mechanism linking uncertainty shocks to macroeconomic variables through firms' financing decisions, with an emphasis on the role of equity financing. When uncertainty is high, equity issuance is limited, as firms are less likely to generate positive profits, and are more tempted to divert profits. As a result, external equity financing shrinks, and this generates additional amplification since total equity financing decreases. Based on this mechanism, I address two questions. First, how are equity financing decisions and associated agency costs affected by uncertainty shocks, and how does equity amplify the response of macroeconomic variables to uncertainty shocks? I build a DSGE financial accelerator model with both debt and equity financing that generates amplification of macroeconomic variables in response to uncertainty shocks. The troughs of macroeconomic variables generated by my model are approximately 30 percent deeper compared to a standard model with only a debt contract. The amplification allows the model to predict procyclical debt and equity financing, and countercyclical external financing costs, a combination which existing models are unable to explain. Second, how does uncertainty affect corporate firms' equity financing decisions empirically? Using balance sheet data of U.S. listed firms from 1993 to 2014, I find that a one standard deviation increase in the level of uncertainty is associated with a 0.7 percentage point decrease in the ratio of equity financing to total assets.

Keywords: Idiosyncratic productivity uncertainty; Debt and equity contract; Financial accelerator; Business cycle amplification; Corporate capital structure; DSGE modeling
JEL codes: E32, E44, G32, D58, D80

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1 Introduction

I develop a dynamic stochastic general equilibrium model where uncertainty shocks are amplified through equity financing frictions. The key prediction of the model is procyclical debt and equity issuance driven by countercyclical costs of debt and equity financing. The model introduces a new transmission mechanism through which uncertainty shocks affect firms' external financing decisions and macroeconomic variables. The financial frictions that arise from equity contracts play a central role in the proposed transmission mechanism. The notion of uncertainty is defined as time-varying dispersion of idiosyncratic productivity across firms (Bloom et al., 2012; Christiano et al., 2014).

In the model, agency problems between entrepreneurs and external shareholders worsen when uncertainty increases. During uncertain times, entrepreneurs are more likely to default and less likely to generate positive profits as lower-tail risk increases. At the same time, entrepreneurs are more tempted to divert profits from external shareholders as upper-tail risk increases. As a result, the agency problem worsens and external shareholders find investing in equity less attractive when uncertainty is high. This limits entrepreneurs' external equity financing as the cost of equity financing increases. Limited equity financing has a direct effect on the size of the balance sheet by reducing total equity. As a consequence, entrepreneurs operate at a smaller size, and a recession ensues.

Based on this mechanism, I answer two research questions. On the theoretical front, I study how equity financing decisions and associated agency costs are affected by uncertainty shocks, and in turn, how equity frictions amplify the response of macroeconomic variables to uncertainty shocks. In particular, I develop a small-scale DSGE financial accelerator model that introduces financial frictions in equity contracts to a standard debt-only model (Carlstrom and Fuerst, 1997). In response to a one standard deviation increase in uncertainty, debt and equity financing decrease from the steady state by approximately 2% and 4% respectively. As firms scale down, aggregate output and investment decrease by approximately 0.4% and 3%, respectively. The troughs of variables in response to uncertainty shocks are approximately 30% deeper than in a standard financial accelerator model with only a debt contract.

The amplification of uncertainty shocks in the model has an important implication for the cyclical properties of debt and equity along with the costs of external financing. In particular, the model simulation generates procyclical debt and equity issuance (Covas and Den Haan, 2011), along with countercyclical costs of debt and equity financing, all of which are consistent with empirical observations. In contrast, existing models are unable explain the coexistence of procyclical debt and equity financing and countercyclical external financing costs. For example, Jermann and Quadrini (2012) build a general equilibrium model to investigate the effect of financial shocks, but their model predicts countercyclical equity financing, which is inconsistent with firm-level evidence reported by Covas and Den Haan (2011). Covas and Den Haan (2012) build a partial equilibrium model to explain procyclical equity financing. However, their model cannot explain the coexistence of procyclical equity financing and countercyclical costs of external financing unless they introduce an ad-hoc assumption on countercyclical costs of equity financing.

The model's key innovation is its explicit modeling of both debt and equity financing decisions. I assume information asymmetry between entrepreneurs and external shareholders, in addition to costly state verification (CSV) (Townsend, 1979), which is a standard debt financing friction. I introduce equity financing frictions following La Porta et al. (2002) and Levy and Hennessy (2007). In particular, I assume that the realization of productivity is entrepreneurs' private information, and entrepreneurs can divert profits from external shareholders by misreporting profits. However, to do so they must sacrifice resources proportional to the size of the balance sheet.

In this environment, entrepreneurs divert profits if and only if realized productivity is sufficiently high so that the size of diverted profits is greater than the cost of diversion. As a result, as upper-tail risk increases, the more entrepreneurs are tempted to divert profits. Since external shareholders internalize the increased probability of profit diversion, equity financing becomes more costly to entrepreneurs as upper-tail risk increases, which limits the amount of equity financing. Equity financing is also limited when lower-tail risk increases, because entrepreneurs are less likely to generate positive profits, so that external shareholders find investing in equity less attractive. A symmetric increase in uncertainty implies increases in both lower- and upper-tail risk. For this reason, the model predicts a decrease in equity financing and an increase in costs of external financing when uncertainty is high.

Within this framework, the response of macroeconomic variables to uncertainty shocks is amplified relative to a model with only debt finance. When uncertainty is high, external equity financing is limited and, in turn, total equity shrinks. This affects the size of the balance sheet both directly and indirectly, as debt financing is further limited, since total equity determines the amount of debt that entrepreneurs can raise.

On the empirical front, I study how firms' equity financing decisions are related to the level of uncertainty, using balance sheet data of U.S. listed firms from annual Compustat for the sample period 1993-2014. Following a panel regression approach suggested by Covas and Den Haan (2011), I find that a one standard deviation increase in the level of uncertainty is associated with a 0.7 percentage point decrease in the ratio of equity financing to total assets, where I measure uncertainty as time-varying dispersion of shocks to firm-level total factor productivity.

In the next section, I discuss related literature and how my work contributes. In Section 3, I introduce debt and equity contracts in a partial equilibrium setting. In Section 4, I embed the partial equilibrium financial contract into a DSGE model. Section 5 presents numerical results of the DSGE model. Section 6 presents empirical evidence on the cyclicality of equity financing in the context of uncertainty shocks. The final section concludes.

2 Literature Review

A wide literature has examined the potential importance of uncertainty shocks as a driver of U.S. business cycles. Among the various channels through which uncertainty affects the macroeconomy, many studies highlight the role of financial frictions. However, these studies mainly focus on debt contracts, and abstract from equity financing frictions.

In contrast, there is a long tradition in the corporate finance literature in which equity financing is not simply a sideshow, for example Myers and Majluf (1984). This literature departs from Modigliani and Miller (1958), in that firms' choice between debt and equity financing has real implications, because it affects the firms' investment decisions.

My research contributes to both strands of the literature. First, I contribute to the literature that studies how uncertainty shocks are transmitted to the economy. Bloom (2009) finds that uncertainty shocks can generate a recession, as firms delay investment until uncertainty is resolved. While Bloom (2009) highlights the "wait-and-see" channel, another line of literature investigates the transmission of uncertainty shocks through financial frictions. For example, Gilchrist et al. (2014) provide evidence that debt frictions play a substantial role in the transmission of uncertainty shocks, and build a DSGE model that is consistent with their empirical findings. In a similar vein, Christiano et al. (2014) estimate a large-scale financial accelerator model with debt contracts and idiosyncratic uncertainty shocks and confirm the significant role of uncertainty shocks in the U.S. business cycle.[1] In this class of models, the default rate increases as uncertainty increases. As a result, debt financing is limited and firms become smaller. Through this channel, adverse uncertainty shocks generate recessions.

However, these studies typically abstract from equity financing. I add to the literature by embedding both debt and equity contracts into the model. Allowing for equity contracts is important, as my model generates a larger amplification of macroeconomic variables to uncertainty shocks compared to models with only debt contracts.

Secondly, I contribute to the literature on the cyclicality of debt and equity financing. For example, Jermann and Quadrini (2006, 2012) document countercyclical equity financing using Flow of Funds Accounts of the Federal Reserve Board, and build a model with both debt and equity financing to study how financial shocks generate business cycles. Their model predicts countercyclical equity financing, which is inconsistent with studies of firm-level data such as Covas and Den Haan (2011, 2012). The latter document that both debt and equity

[1]Similarly Cesa-Bianchi and Fernandez-Corugedo (2015) build a DSGE model with both macro-level uncertainty (time-varying second moment of TFP shocks) and micro-level uncertainty (time-varying dispersion of idiosyncratic productivity) and show that micro-level uncertainty shocks generate a recession through financial market frictions.

financing are procyclical for listed U.S. firms in all size classes except for the top 1% firms by asset size, where smaller firms have stronger procyclicality. Covas and Den Haan (2012) develop a partial equilibrium model in which firms finance investment with both debt and equity. In their model, firms scale up their business in response to positive productivity shocks. However, since debt financing increases the likelihood of default, firms have an incentive to issue equity to avoid excessive leverage when they issue debt.[2] Although their model predicts both procyclical debt and equity financing, it fails to predict countercyclical real borrowing costs and a countercyclical default rate unless countercyclical equity financing costs are assumed. They introduce countercyclical equity financing costs into the model by simply assuming an ad-hoc functional relationship between productivity and equity financing costs without a microfoundation.

In contrast, my framework predicts procyclical debt and equity financing along with endogenous countercyclical external financing costs, all of which are consistent with the data. Under the CSV framework, agency costs decrease when the level of TFP decreases since firms scale down and need less external financing. While adverse uncertainty shocks partly offset this effect by increasing agency costs, the effect of adverse uncertainty shocks is not large enough to generate countercyclical agency costs, if only a debt contract is considered. However, in my model with both debt and equity contracts, the effect of uncertainty shocks on the cyclicality of financing frictions dominates the effect of TFP level shocks, as equity financing frictions amplify uncertainty shocks. So, the model is able to generate both procyclical debt and equity financing, a countercyclical default rate and countercyclical cost of debt and equity finance.

My empirical finding that equity financing is negatively correlated with uncertainty is related to existing empirical studies of the patterns and cyclicality of debt and equity financing. Fama and French (2005) document that equity financing is common among listed firms

[2]Begenau and Salomao (2015) build a heterogenous agent general equilibrium model to simultaneously explain procyclical equity financing of small firms and coutercyclicality of the largest firms. Small firms issue debt and equity procyclically for a similar reason as in Covas and Den Haan (2012). However, the largest firms find debt financing much cheaper during the expansion, since they are already close to the efficient scale of production, and thus the impact of having an additional unit of debt on the default probability is low. As a result, they replace equity with debt during expansions to take an advantage of a tax benefit on debt over equity.

in the U.S. Covas and Den Haan (2011, 2012) show that both debt and equity financing are procyclical for listed U.S. firms in all size classes except for the top 1% of firms by asset size, where smaller firms have stronger procyclicality. Erel et al. (2012) document a similar pattern. They find that seasoned equity offerings (SEO) decrease during NBER-defined recessions, which is a pattern largely driven by noninvestment-grade firms. They also find that bond financing is procyclical, which is also largely driven by noninvestment-grade firms. I add to this literature by investigating cyclical patterns of debt and equity finance in response to changes in uncertainty. I provide empirical evidence that debt and equity financing decreases during periods of high uncertainty, and build a DSGE model that is consistent with empirical evidence.

3 Financial Contracts

In this section, I build a theoretical model with both debt and equity contracts that predicts a decrease in debt and equity financing in response to increased uncertainty. I first discuss the financial contract among entrepreneurs, lenders, and external shareholders in a partial equilibrium setting. The partial equilibrium analysis will be extended to a dynamic stochastic general equilibrium model in Section 4.

I model the debt contract as in Carlstrom and Fuerst (1997), who introduce debt financing frictions into a computationally tractable general equilibrium model. The main friction in the debt contract arises from an information asymmetry between lenders and borrowers. Following Townsend (1979), lenders must pay a monitoring cost in order to verify the true productivity of borrowers (Costly State Verification, CSV). However their model abstracts from equity financing. I introduce equity financing frictions into Carlstrom and Fuerst (1997) by assuming that entrepreneurs can divert profits at some cost, following La Porta et al. (2002) and Levy and Hennessy (2007).

Three types of agents participate in the financial contract: entrepreneurs, lenders, and external shareholders. I assume that all contract parties are risk neutral, and only care about expected returns. Entrepreneurs, who operate capital good producing firms, have access to a stochastic constant-returns-to-scale capital production technology which transforms consumption goods into capital goods. Entrepreneurs finance their investment projects prior to the realization of idiosyncratic productivity shocks using debt and external equity, along with internal equity. Most of the financial accelerator literature abstracts from external equity and uses the term 'net worth' to refer to both total equity and internal equity. However, there is a clear distinction between internal and external equity in this paper. To avoid confusion, I use the term internal equity instead of net worth to refer to the funds that entrepreneurs put into the contract. After the realization of idiosyncratic productivity, entrepreneurs can potentially either default on debt or divert profits. In case of debt default, lenders pay a monitoring cost to verify the realized idiosyncratic shock and take all the output from entrepreneurs. In case of profit diversion, entrepreneurs first repay principal and interest on debt. However, instead of paying all remaining profits to external shareholders, entrepreneurs take a fraction of the profit which should belong to external shareholders. To divert profits entrepreneurs must sacrifice a certain amount of capital goods, which is proportional to the size of the balance sheet.

There are two sources of aggregate risk in the economy: total factor productivity (TFP) shocks, and uncertainty shocks. TPF shocks are standard as in real business cycle models. Uncertainty shocks refer to a stochastic time-varying dispersion of idiosyncratic productivity shocks. Aggregate shocks are realized at the beginning of the period. All financial contracts are intra-temporal, and thus there is no aggregate shock realized for the duration of the contract. As a result all parties take the dispersion of idiosyncratic productivity shocks parametrically. This assumption further allows us to analyze the model first in partial equilibrium, and then in a dynamic general equilibrium setting.

Lenders and external shareholders can be thought of as financial institutions that channel funds from households (which I will discuss when I describe the DSGE model) to entrepreneurs who produce capital goods. The economy is populated with numerous infinitesimal lenders and external shareholders that specialize either in debt or equity. Each financial institution pools deposits from households and lends to or buys shares of numerous infinitesimal entrepreneurs. This allows financial institutions to diversify idiosyncratic risks, and guarantee a fixed return to households.

3.1 Setup of Debt and Equity Contracts

I now, analyze debt and equity contracts in a partial equilibrium setting. An entrepreneur i has access to a stochastic constant returns to scale technology $\omega_i i_i$, which transforms i_i units of consumption goods into $\omega_i i_i$ units of capital goods, taking the price of capital q as given.[3] The consumption good is the numeraire, and the price of capital will be endogenously determined in general equilibrium. ω_i denotes an idiosyncratic productivity shock whose distribution is $\ln(\omega_i) \sim N(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2)$.[4] Idiosyncratic productivity shocks are independently and identically distributed across entrepreneurs in each period. $\phi(\omega)$ and $\Phi(\omega)$ denote the p.d.f and c.d.f of ω_i respectively. The modeling of uncertainty closely follows Christiano et al. (2014). Uncertainty shocks are embedded into the model by assuming that the dispersion of idiosyncratic productivity shocks σ_ω is a time-varying stochastic variable. However, σ_ω does not vary within the duration of financial contracts. For this reason, contract participants take σ_ω parametrically in a partial equilibrium setting. Since contracts are intra-period, I suppress time subscript t for notational simplicity.

An entrepreneur has three different sources for financing their investment project i_i. The investment project requires consumption goods. The first option is to finance with internal equity n_i. For now, I assume that n_i is exogenously fixed for the duration of the contract. In general equilibrium, n_i is determined endogenously as a result of entrepreneurs' capital accumulation decisions. The second option is to issue debt securities $d_i = i_i - e_i - n_i$, where e_i denotes the third source of financing, which is external equity. The size of the project i_i also represents the size of the balance sheet of a capital good producing firm, which consists of debt and total equity, the latter of which in turn is a sum of internal and external equity.[5]

I limit my interest to the case where the optimal size of project i_i is greater than internal equity n_i so that all firms must rely on external financing to some degree. Entrepreneurs

[3] From now on, I use entrepreneurs and capital good producing firms interchangeably depending on context.

[4] This implies $E(\omega_i) = 1$.

[5] From now on, I use the size of the project and the size of the balance sheet interchangeably depending on context.

borrow d_i before idiosyncratic productivity is realized and promise to return $(1 + r_d)d_i$ units of capital goods to lenders once idiosyncratic productivity is realized and production is taken. After they observe the realization of idiosyncratic productivity, capital good producing firms can default on debt. The realization of idiosyncratic productivity is the entrepreneur's private information. Due to the information asymmetry between lenders and borrowers, lenders have to sacrifice μi_i units of capital goods to verify the firm's reports in case of default. This is a CSV framework as in Carlstrom and Fuerst (1997). After lenders pay the monitoring cost, they seize $\omega_i i_i$, which will in equilibrium be less than the sum of principal and interest.

The other source of external financing is outside equity e_i raised from external shareholders. Entrepreneurs raise equity before the realization of idiosyncratic productivity, and promise to return the fraction $s_i \in [0, 1]$ of the profit (in capital good units) to shareholders as dividends once idiosyncratic productivity is realized. However entrepreneurs can divert profits at a cost proportional to the size of the balance sheet, γi_i. The greater is γ, the more costly it is for entrepreneurs to divert profits since they sacrifice a larger fraction of their balance sheet in case of diversion. Thus, a higher γ represents an economy with better outside investor protection. If profit diversion occurs, entrepreneurs repay their debt to lenders and take a fraction ϕ of the portion of profits promised to shareholders, plus all of the profits promised to themselves, net of diversion costs. ϕ is parametrically given and measures the degree of investor protection together with γ. The higher is ϕ, the more entrepreneurs can divert from external shareholders as the degree of investor protection is low. The return to entrepreneurs under diversion thus equals $(1-s_i)\left[\omega_i i_i - (1 + r_d)d_i\right] + \phi s_i \left[\omega_i i_i - (1 + r_d)d_i\right] - \gamma i_i$. Note that due to the information asymmetry, external shareholders cannot verify the realized value of idiosyncratic productivity. By assumption, they do not have access to a CSV technology. The friction embedded in the equity contract is taken from La Porta et al. (2002) and Levy and Hennessy (2007). In practice, profit diversion can occur in various forms both legally and illegally. For example, entrepreneurs might reward themselves with excessively large salaries, or install unqualified family members in managerial positions. They could also divert profits by benefiting outside entities controlled by the entrepreneurs, for example, by providing better terms of contract or by transferring assets. In the worst case, entrepreneurs can simply steal profits. In this regard, entrepreneurs in my model represent any type of manager, controlling shareholder, and/or board member who owns a share of the firm's assets, and at the same time actively engages in the firm's managerial decisions. See Johnson et al. (2000) and La Porta et al. (2000) for an extensive list of profit diversions that could occur in practice.

Lastly, note that financing decisions are made prior to the realization of idiosyncratic productivity shocks. As a result, adverse selection is not present in this environment. For example, it will not be the case that firms intending to divert funds are the only firms active in the equity market.

3.2 Debt Default and Asset Diversion Thresholds

The next step is to find the productivity thresholds for default and diversion. Entrepreneurs default on debt only if they cannot repay promised returns to lenders. There is no incentive to default when realized output is greater than the sum of the principal and interest, because lenders can recoup their claims in this case. Therefore firms default if and only if the realized shock ω_i satisfies $\omega_i i_i < (1 + r_d)d_i$. Thus, the debt default threshold $\bar{\omega}_i$ is defined as

$$
\begin{aligned}
\bar{\omega}_i &\equiv \frac{(1 + r_d)d_i}{i_i} \\
&= \frac{(1 + r_d)(i_i - e_i - n_i)}{i_i}
\end{aligned}
$$

For any given level of external equity e_i and internal equity n_i, once capital good producing firms decide on the debt default threshold $\bar{\omega}_i$ and the size of project i_i, the corresponding interest rate r_d is determined by $(1 + r_d) = \frac{\bar{\omega}_i i_i}{(i_i - e_i - n_i)}$. Also note that the share s_i of profits promised to external shareholders does not affect the properties of the debt contract, because shares are residual claims.

Meanwhile entrepreneurs divert profits if and only if the payoff from diversion is greater than the payoff from honoring the equity contract. Therefore entrepreneurs divert profits when the realized shock satisfies

$$
\begin{aligned}
(1-s_i)\left[\omega_i i_i - (1+r_d)d_i\right] \quad < \quad & (1-s_i)\left[\omega_i i_i - (1+r_d)d_i\right] \\
& + \phi s_i \left[\omega_i i_i - (1+r_d)d_i)\right] - \gamma i_i.
\end{aligned}
$$

The right-hand side of the inequality denotes the payoff in case of diversion while the left-hand side denotes the payoff in case of honoring the equity contract. By replacing $(1+r_d)d_i$ on both sides with $\bar{\omega}_i i_i$, as derived, this inequality simplifies to

$$
\omega_i > \bar{\omega}_i + \frac{\gamma}{\phi s_i}.
$$

The right-hand side of this inequality defines the profit diversion threshold productivity $\widehat{\omega}_i \equiv \bar{\omega}_i + \frac{\gamma}{\phi s_i}$. Entrepreneurs divert profits when the realized shock is above $\widehat{\omega}_i$. First, note that $\widehat{\omega}_i$ is an increasing function of γ, the diversion cost, and a decreasing function of ϕ, the share of profits the firm can divert. These results are straight forward. Also, $\widehat{\omega}_i$ is increasing in the fraction of external shares, s_i. A higher fraction of external shares implies that entrepreneurs are only entitled to a small fraction of the profit. In such case, entrepreneurs are more willing to engage in diversion so that they can seize the portion that otherwise belongs to the external shareholders. Second, profit diversion occurs only when the realization of ω_i is sufficiently large. This result is intuitive, since profit diversion is not optimal if entrepreneurs receive nothing after paying the cost of diversion. This result is consistent with Levy and Hennessy (2007), whose model predicts a stronger incentive for diversion when the realized profits are sufficiently large. Third, the ratio between γ and ϕ (along with s_i) is sufficient to determine the profit diversion threshold. Considering that both parameters measure the degree of investor protection, having both parameters separately might seem redundant. However, each parameter plays a unique role in the model. γ affects the amount of deadweight loss due to diversion, and directly affects entrepreneurs' payoff. Meanwhile, external shareholders' payoff is directly affected by the level of ϕ. Finally, note that $\widehat{\omega}_i > \bar{\omega}_i$. This implies capital good producing firms do not have any incentive to default when they conduct diversion. This is obvious because if there is no profit, there are no resources to divide.

3.3 Equilibrium Contract

Since entrepreneurs, lenders, and shareholders are risk neutral during the financial contract period, the expected payoff is the only concern to each party. In this environment, entrepreneurs will choose $(\bar{\omega}_i,\, i_i,\, e_i,\, s_i)$ so that their expected payoff is maximized, subject to both lenders and shareholders earning an expected gross return of one. Considering that the financial contracts are intra-period, an expected gross return of one is sufficient to ensure lenders' and external shareholders' participation.

Entrepreneurs' expected payoff from participating in the contract is

$$\begin{aligned}
\textit{Expected Payoff (entrepreneur)} &= \int_{\bar{\omega}}^{\widehat{\omega}} (1-s)\left[\omega i-(1+r_d)(i-e-n)\right]\Phi(d\omega) \\
&\quad + \int_{\widehat{\omega}}^{\infty} \{(1-s)\left[\omega i-(1+r_d)(i-e-n)\right] \\
&\quad + \phi s\left[\omega i-(1+r_d)(i-e-n)\right]-\gamma i\}\Phi(d\omega) \\
&= \int_{\bar{\omega}}^{\infty}(1-s)\left[\omega i-\bar{\omega} i\right]\Phi(d\omega)+\int_{\widehat{\omega}}^{\infty}\{\phi s\left[\omega i-\bar{\omega} i\right]-\gamma i\}\Phi(d\omega) \\
&= i\left[\int_{\bar{\omega}}^{\infty}(1-s)\left[\omega-\bar{\omega}\right]\Phi(d\omega)+\int_{\widehat{\omega}}^{\infty}\{\phi s\left[\omega-\bar{\omega}\right]-\gamma\}\Phi(d\omega)\right] \\
&= i\times A(\bar{\omega},\, s)
\end{aligned}$$

where $A(\bar{\omega},\, s) \equiv \int_{\bar{\omega}}^{\infty}(1-s)(\omega-\bar{\omega})\Phi(d\omega)+\int_{\bar{\omega}+\frac{\gamma}{\phi s}}^{\infty}(\omega-\bar{\omega}-\gamma)\,\Phi(d\omega)$ denotes the expected share of output (in terms of capital good units) paid to entrepreneurs.[6] The first term of the first line of the equation denotes entrepreneurs' expected payoff when paying dividends to external shareholders truthfully, while the second and the third terms show the expected payoff to entrepreneurs in case of profit diversion.

[6] I drop subscript i for notational simplicity.

The expected payoff to lenders is

$$\begin{aligned}
Expected\ Payoff\ (lender) &= \int_0^{\bar{\omega}} \omega i \Phi(d\omega) - \Phi(\bar{\omega})\mu i + [1-\Phi(\bar{\omega})]\,(1+r_d)(i-e-n) \\
&= \int_0^{\bar{\omega}} \omega i \Phi(d\omega) - \Phi(\bar{\omega})\mu i + [1-\Phi(\bar{\omega})]\,\bar{\omega} i \\
&= i\left[\int_0^{\bar{\omega}} \omega \Phi(d\omega) - \Phi(\bar{\omega})\mu + [1-\Phi(\bar{\omega})]\,\bar{\omega}\right] \\
&= i \times B(\bar{\omega},\, s)
\end{aligned}$$

where $B(\bar{\omega},\, s) \equiv \int_0^{\bar{\omega}} \omega \Phi(d\omega) - \Phi(\bar{\omega})\mu + [1-\Phi(\bar{\omega})]\,\bar{\omega}$ denotes the expected share of output (in terms of capital good units) paid to lenders. The first two terms of the first line of the equation denote lenders' expected payoff when firms default on debt while the last term shows the payoff when firms experience sufficiently large productivity that they repay lenders in full. Note that equity share s is not present in the expression $B(\bar{\omega},\, s)$. Since lenders are always repaid with highest priority, s does not directly affect the share of output that lenders will receive. However, s does affect the expected share of output paid to lenders indirectly, since there is an interdependence between $\bar{\omega}$ and s.

Similarly, external shareholders' expected payoff is

$$\begin{aligned}
Expected\ Payoff\ (shareholder) &= \int_{\bar{\omega}}^{\infty} s\left[\omega i - (1+r_d)(i-e-n)\right]\Phi(d\omega) \\
&\quad - \int_{\widehat{\omega}}^{\infty} \phi s\left[\omega i - (1+r_d)(i-e-n)\right]\Phi(d\omega) \\
&= \int_{\bar{\omega}}^{\infty} s\left[\omega i - \bar{\omega} i\right]\Phi(d\omega) - \int_{\widehat{\omega}}^{\infty} \phi s\left[\omega i - \bar{\omega} i\right]\Phi(d\omega) \\
&= i\left[\int_{\bar{\omega}}^{\infty} s\left[\omega - \bar{\omega}\right]\Phi(d\omega) - \int_{\widehat{\omega}}^{\infty} \phi s\left[\omega - \bar{\omega}\right]\Phi(d\omega)\right] \\
&= i \times C(\bar{\omega},\, s)
\end{aligned}$$

where $C(\bar{\omega},\, s) \equiv \int_{\bar{\omega}}^{\infty} s\left[\omega - \bar{\omega}\right]\Phi(d\omega) - \int_{\bar{\omega}+\frac{\gamma}{\phi s}}^{\infty} \phi s\left[\omega - \bar{\omega}\right]\Phi(d\omega)$ denotes the expected share of output (in terms of capital good units) paid to shareholders.

The sum of the expected shares paid to each party is given by

$$\begin{aligned}A+B+C &= \int_0^\infty \omega\Phi(d\omega) - \Phi(\bar{\omega})\mu - \left[1-\Phi\left(\bar{\omega}+\frac{\gamma}{\phi s}\right)\right]\gamma \\ &= 1-\Phi(\bar{\omega})\mu - \left[1-\Phi\left(\bar{\omega}+\frac{\gamma}{\phi s}\right)\right]\gamma\end{aligned}$$

where the second term denotes the expected loss due to the monitoring cost, and the third term denotes the expected loss due to the diversion cost. Note that the expected shares paid to each party do not add up to 1, because output may be lost due to costly monitoring or diversion.

Given these expressions for the expected payoffs to each party, the contract problem is defined as

$$\max_{\bar{\omega},s,i,e} qiA(\bar{\omega},s) \quad \textit{subject to} \quad \begin{aligned} qiB(\bar{\omega},s) &\geqq i-e-n \\ qiC(\bar{\omega},s) &\geqq e \end{aligned}$$

where q is the price of capital goods. Entrepreneurs maximize their expected payoff in consumption goods units by optimally choosing $(\bar{\omega},\ s,\ i,\ e)$. The entrepreneurs' objective function is expressed in terms of consumption goods, since entrepreneurs utility depends on consumption in the general equilibrium model presented in Section 4. Furthermore lenders participate in the contract only if they recoup the resources they lend in expectation, and external shareholders accept the equity contract only if they receive expected returns at least as large as the amount of external equity they provide to entrepreneurs. Note that the price of capital good q appears on the left-hand side of both constraints but not on the right-hand side, since both debt and equity are raised in consumption goods, and entrepreneurs pay back in capital goods. Obviously both constraints bind with equality at an optimum. From entrepreneurs' perspective, for a given level of external financing (d and e), entrepreneurs want to minimize the fractions of output paid to lenders $(B(\bar{\omega},\ s))$ and to shareholders $(C(\bar{\omega},\ s))$ so that firms can receive a higher fraction of the output.

Note that I make a simplifying assumption that debt and equity investors' behavior is passive. Debt and equity investors are not making an optimal portfolio decision between debt and equity. Instead, equity investors commit to invest only in equity but not in debt, and vice versa for debt investors. In this regard, the contract is optimal only from firms' perspective, and the optimal contract might be different in an environment where investors make an optimal portfolio decision between debt and equity.

Solving the financial contract problem, the optimality conditions are given by

$$
\begin{aligned}
\frac{A_1}{A_2} &= \frac{B_1 + C_1}{B_2 + C_2} && (1)\\
q &= \frac{1}{(B + C) - \frac{(B_1+C_1)}{A_1} A} && (2)\\
i &= \frac{1}{1 - q(B + C)} n && (3)\\
e &= \frac{qC}{1 - q(B + C)} n && (4)
\end{aligned}
$$

where A_1, B_1, and C_1 denote partial derivatives of A, B, and C with respect to $\bar{\omega}$, while A_2, B_2 and C_2 denote partial derivatives with respect to s. The interpretation of these optimality conditions is similar to that of Carlstrom and Fuerst (1997). For completeness, I repeat their interpretation. The first important observation is that for any given price of capital q, equations (1) and (2) pin down $\bar{\omega}$ and s. Also, note that the optimal $\bar{\omega}$ and s depend implicitly on q, but are independent of the level of internal equity. As a result, all entrepreneurs have identical s and $\bar{\omega}$, and thus the expected shares paid to each party $A(\omega, s)$, $B(\omega, s)$, and $C(\omega, s)$ are identical across entrepreneurs. Substituting this result into equations (3) and (4), the size of the project i and external equity e are defined as functions of q and n. Rewriting the solution of equation (3) as $i(q, n)$, and aggregating $\omega i(q, n)$ across entrepreneurs, the law of large numbers implies an aggregate investment good supply function $I^S(q, n) \equiv i(q, n) \left\{1 - \Phi(\bar{\omega})\mu - \left[1 - \Phi\left(\bar{\omega} + \frac{\gamma}{\phi s}\right)\right] \gamma\right\}$, where n is an average (or aggregate) of individual internal equity across entrepreneurs, with a slight abuse of notation. Thus aggregate investment is a function solely of the economy-wide capital price q, and aggregate internal equity n. The linearity of the firms' balance sheet in internal equity is a direct consequence of the assumption that the costs of state verification and profit diversion are linear in the size of project i. Without linearity, the computational burden would increase substantially once the partial equilibrium contract is embedded into a DSGE setting, since it would be necessary to track the distribution of internal equity to solve the model.

The second important observation concerns the expected return on internal saving $\frac{qAi}{n}$. Replacing i with equation (3), $\frac{qAi}{n}$ is equal to $\frac{qA}{1-q(B+C)}$. This term is important in understanding the evolution of entrepreneurs' internal equity in a DSGE setting. In the absence of financial frictions, the expected return on internal saving is always equal to one, which implies that returns on debt, external equity and internal equity are identical.[7] Consistent with the Modigliani-Miller theorem, this further implies that the financing method is irrelevant to entrepreneurs. However, as long as financial frictions are present, there is a deadweight loss from external financing due to the costly state verification and profit diversion. Since lenders, external shareholders, and entrepreneurs internalize the loss, the expected return on internal saving is always greater than one, and from the above expression for $\frac{qAi}{n}$ the size of the expected return depends on the level of debt and external equity.[8] As a result financial structure does matter, and this incentivizes entrepreneurs to adjust internal equity accordingly over time in the DSGE model discussed in Section 4.

Lastly, entrepreneurs rely on *both* debt and external equity in equilibrium. In other words, it is not optimal for entrepreneurs to use a single source of external finance. Consider an entrepreneur who finances completely through debt. Intuitively, marginally increasing external equity barely affects the probability of diversion. This implies that external shareholders will not ask for a high premium for buying shares. As a result entrepreneurs will replace debt with equity. Now, consider the opposite case where entrepreneurs finance their project entirely with equity. In this case, marginally increasing debt barely increases the default probability, and this implies a low real borrowing cost. As a result entrepreneurs will

[7] q is greater than 1 in equilibrium if financial frictions are present. This essentially reflects a compensation to contractual parties for participating in debt and equity contracts which incur deadweight loss. As a result $q = 1$ in the absence of financial frictions. At the same time $A + B + C = 1$ without financial frictions. Substituting $q = 1$ and $A = 1 - (B + C)$ into $\frac{qA}{1-q(B+C)}$ yields $\frac{qA}{1-q(B+C)} = 1$.

[8] Note that A, B, and C are functions of s and $\bar{\omega}$, and equations (1)-(4) pin down $\bar{\omega}$, s, i, and e.

replace equity with debt.

3.4 Outcome of Financial Contracts

In this section, I present numerical results on the effect of changes in uncertainty on financial contracts in a partial equilibrium setting. In this case, in a partial equilibrium setting, contract parties take the capital price q and internal equity n as fixed. I conduct a comparative statics exercise by changing the value of the dispersion of idiosyncratic productivity σ_ω while holding the capital price q and other parameters fixed, for a given level of internal net worth n. Parameter values used in this exercise are reported under the heading "Financial Friction" in Table 1. They are chosen based on a calibration of the DSGE model, which will be discussed in Section 4.

Figure 1 shows changes in the levels of balance sheet variables for different values of σ_ω. Solid lines represent percentage deviations from the contract outcome under a baseline parameterization (with dispersion of idiosyncratic productivity $\sigma_\omega = 0.2466$) when both debt and equity contract frictions are present. If the level of uncertainty increases (higher values of σ_ω), entrepreneurs raise less external equity as shown in the top-left panel of Figure 1. This result is consistent with empirical evidence, reported in Section 6, that increased uncertainty is associated with a decrease in equity financing. Furthermore, entrepreneurs scale down the level of debt (top-right panel of Figure 1) in response to increased uncertainty. As a consequence, the size of the project shrinks (bottom-left panel of Figure 1).

What is the underlying mechanism that drives firms to lower both debt and equity when uncertainty increases? Regarding the debt contract, as uncertainty increases, the probability of default increases for any given level of total equity. As a result, lenders find debt securities less attractive, and the lenders' demand for debt decreases. Entrepreneurs find debt financing more expensive since they must compensate lenders for bearing a higher default probability. As a consequence, debt financing decreases in equilibrium.

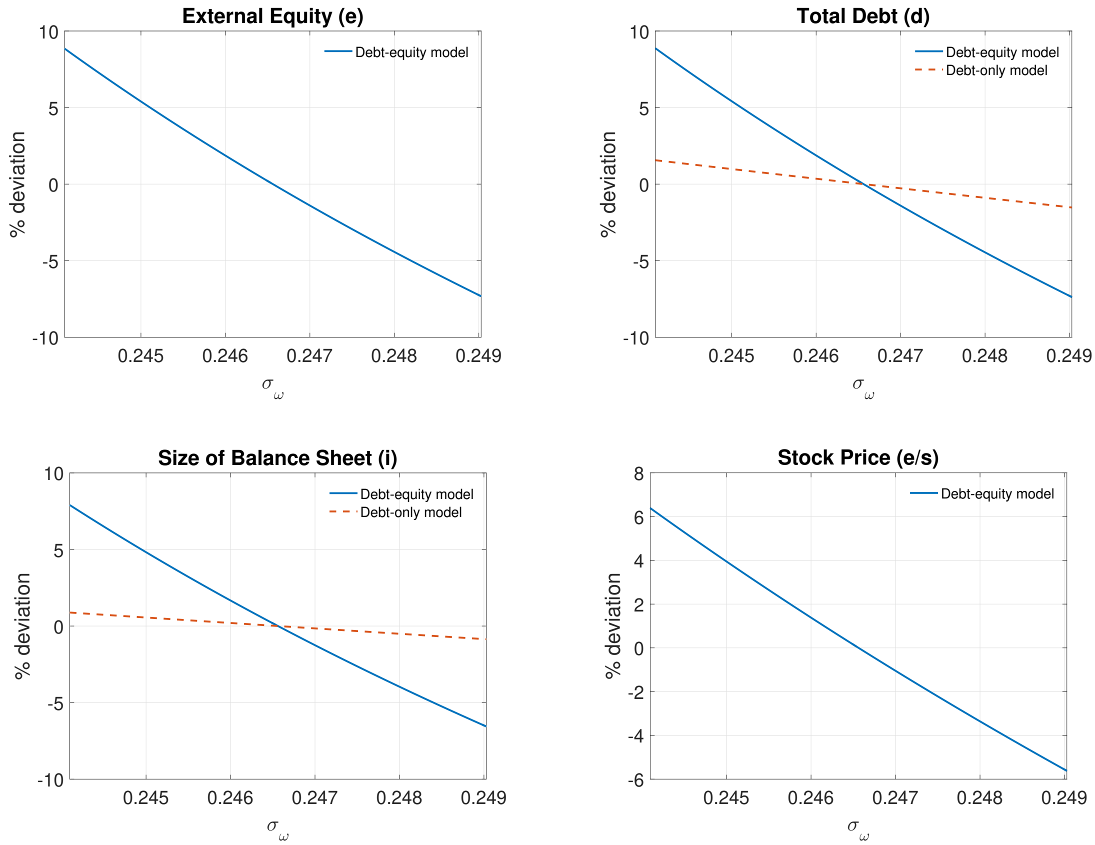
The model predicts a decrease in equity financing when uncertainty increases, for two reasons. First, for any given level of internal equity n, external equity e, and debt d, the probability of default increases as the level of uncertainty increases, since lower tail risk increases.

Table 1: Calibration

Parameter	Description	Debt-only model	Debt-equity model	Target
Preference				
β	Discount factor	0.99	0.99	Standard RBC
ξ	Additional entrepreneurial discount factor	0.9453	0.9084	real borrowing cost $q(1+r_d)-1$ = 60 bps
ν	Labor disutility	2.8077	2.8049	h = 0.3
Production				
α	Capital elasticity	0.36	0.36	Standard RBC
κ	Labor elasticity	0.6399	0.6399	Standard RBC
δ	Depreciation rate	0.02	0.02	Standard RBC
Financial Friction				
μ	Costly state verification	0.25	0.25	CF (1997)
γ	Costly profit diversion	-	0.1682	$1-s$ = 0.26
ϕ	Fraction diverted	-	0.2296	Default rate $\Phi(\bar{\omega})$ = 1.12%
$\sigma_{\omega,ss}$	Steady state dispersion of idiosyncratic productivity shocks	0.2466	0.2466	Estimated
Aggregate Shocks				
ρ_θ	Persistence of TFP shocks	0.78	0.78	Estimated
ρ_{σ_ω}	Persistence of uncertainty shocks	0.83	0.83	Estimated
σ_θ	S.D of TFP shocks	0.0071	0.0071	Estimated
σ_{σ_ω}	S.D of uncertainty shocks	0.0469	0.0469	Estimated
$corr\left(\epsilon_{\sigma_{\omega,t}}, \epsilon_{\theta,t}\right)$	Correlation between two shocks	-0.5887	-0.5887	Estimated
Other Parameters				
η	Entrepreneurial mass	0.02	0.02	CF (1997)

Notes: The table shows calibration results of the debt-only model and the debt-equity model. CF (1997) refers to Carlstrom and Fuerst (1997).

Figure 1: Partial Equilibrium Analysis - Changes in σ_ω



Notes: The figure shows partial equilibrium contract outcomes for different values of σ_ω. All values are percentage deviations from the contract outcomes calculated at the baseline parameterization ($\sigma_\omega = 0.2466$). Solid lines are contractual outcomes from the debt-equity model. Dashed lines are contractual outcomes from the debt-only model. See Table 1 for values of other parameters.

This implies that it is less likely for entrepreneurs to generate positive profits and dividends. From the external shareholders' perspective, investing in equity becomes less attractive, and as a consequence, shareholders demand equity less. Second, investing in equity is less attractive due to increased upper tail risk. As discussed in the previous section, entrepreneurs are more tempted to divert profits if the realization of the idiosyncratic productivity shock is high. Internalizing the increased chance of profit diversion, shareholders demand equity less. As a result, equity financing decreases in equilibrium. The bottom-right panel of Figure 1 shows how external equity per share, or equivalently the price of stock (e/s), varies for different values of σ_ω. It is clear that equity financing becomes more expensive from entrepreneurs' perspective when uncertainty increases, which limits the amount of equity that entrepreneurs can raise.

Answering how firms' capital structure and financing decisions vary in response to uncertainty is itself an important question in corporate finance. However, we are also interested in the macroeconomic consequences of increased uncertainty when equity financing is explicitly taken into account. The most important macroeconomic implication of the model is an *amplification* of uncertainty shocks through the equity financing friction. The amplification of uncertainty arising from equity financing frictions can be shown by comparing contract outcomes in the model with both debt and equity (hereafter the debt-equity model), and the model with only debt (hereafter the debt-only model).

The dashed lines in Figure 1 represent percentage deviations from the contract outcome under a baseline parameterization ($\sigma_\omega = 0.2466$) of the debt-only model, which is exactly identical to Carlstrom and Fuerst (1997).[9] As the bottom-left panel of Figure 1 shows, the size of the balance sheet responds more to uncertainty in the debt-equity model than in the debt-only model. The amplification arises mainly from the fact that total equity includes both internal equity and external equity in the debt-equity model, but only internal equity in the debt-only model. Since external equity decreases as a result of increased uncertainty, total equity shrinks in the debt-equity model. However, total equity remains constant in the debt-only model. In addition, since total equity determines the debt capacity, shrinking total equity further limits debt financing in the debt-equity model. As the top-right panel of Figure 1 suggests, debt financing shrinks more in the debt-equity model when uncertainty increases.

In a general equilibrium setting, internal equity will vary over time in both models, as entrepreneurs adjust internal savings, which form the internal equity of following periods. However, since the debt-equity model has the additional margin of external equity financing, total equity is expected to exhibit larger fluctuations in response to uncertainty shocks

[9] An alternative way to shut down equity financing is to set $\gamma \to 0$, so that profit diversion is virtually costless. In this case, entrepreneurs will always divert profits regardless of the size of profit. Since shareholders internalize the fact that profit diversion always occurs, they will never invest in equity. As a result, the equity market collapses.

in following periods as well.

De Fiore and Uhlig (2011, 2015) investigate the role of bond and bank loan financing over the business cycle. They show that an economy with a well-developed bond market (along with a bank loan market) is less vulnerable to adverse shocks than an economy heavily dependent on the bank loan market (with a less developed bond market), since firms can substitute one from the other in response to shocks. In contrast, my model predicts that having an additional source of external financing amplifies shocks. The different prediction is mainly due to the complementarity between debt and equity. The amount of equity determines the amount of debt a firm can raise in my model. However, there is no such relationship between bank loans and bond financing in De Fiore and Uhlig (2011, 2015), who focus on substitutability between the two types of debt instruments.

4 General Equilibrium Analysis

4.1 Setup of the Model

In this section, I embed the partial equilibrium financial contract into a dynamic stochastic general equilibrium model. The main goal of this section is to investigate the dynamic effects of uncertainty shocks on financing decisions and macroeconomic outcomes. In contrast to the partial equilibrium analysis, the price of capital goods q and internal equity n are determined endogenously in equilibrium. The model closely follows Carlstrom and Fuerst (1997). The major differences are introducing the equity contract and uncertainty shocks, in the form of a time-varying stochastic dispersion of idiosyncratic productivity.

The economy is populated with a unit mass continuum of economic agents. There are two types of agents in the model: households with fraction $1 - \eta$ and entrepreneurs with fraction η. Households are standard as in conventional real business cycle models. However, entrepreneurs are non-trivial. They have an access to a stochastic technology which transforms consumption goods into capital goods. The role of entrepreneurs is critical in the model since entrepreneurs are subject to financial frictions when they finance input costs of capital production. Entrepreneurs finance input costs through financial institutions that pool households' funds and invest in debt and equity. Consumption good producing firms are standard. They take labor and capital as inputs and are not subject to financial frictions. Households are infinitely-lived and risk averse. They maximize expected lifetime utility by optimally choosing consumption c_t^h and leisure l_t where the time endowment is normalized to unity. They discount the future utility with time preference parameter $\beta \in (0, 1)$. Since there is no heterogeneity across households, I study a representative household hereafter. Households accumulate physical capital k_t^h, which earns gross interest $1 + r_t$ and depreciates at the rate δ in the following period. They also earn wage income by supplying labor to consumption good producing firms, at a wage rate w_t. They purchase consumption goods at a price of unity (the consumption good is the numeraire), and they also purchase new capital goods at the end of the period at a price of q_t. The representative household's utility maximization problem at time 0 is formally given as follows:

$$\max_{c_t^h, k_{t+1}, h_t} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^h,\, 1 - h_t\right)$$

subject to

$$c_t^h + q_t k_{t+1} \leqq w_t h_t + r_t k_t + q_t(1-\delta)k_t.$$

The maximization problem yields the following standard intratemporal and intertemporal optimality conditions:

$$w_t = \frac{u_L(t)}{u_c(t)} \tag{5}$$

$$q_t u_c(t) = \beta E\left[u_c(t+1)\left\{r_{t+1} + q_{t+1}(1-\delta)\right\}\right]. \tag{6}$$

Identical consumption good producing firms owned by households have access to a constant-returns-to-scale technology given by $Y_t = \theta_t F(K_t, H_t, H_t^e)$. They produce consumption goods Y_t taking the aggregate capital stock K_t, aggregate household labor H_t, and aggregate entrepreneurial labor H_t^e as inputs. The technology is subject to aggregate TFP shocks θ_t, realized at the beginning of period t. Consumption good producing firms are price takers in both input and output markets. Solving their profit maximization problem yields standard capital, household labor, and entrepreneurial labor demand curves given by

$$
\begin{aligned}
r_t &= F_K(t) \\
w_t &= F_H(t) \\
w_t^e &= F_{H^e}(t).
\end{aligned}
$$

In the baseline calibration (see Section 4.3 and Table 1), entrepreneurial labor plays a minimal role in the consumption good production process. However, it is important to include entrepreneurial labor since it allows entrepreneurs to start a new business with non-zero internal equity in case of default. If entrepreneurial labor is not included, entrepreneurs start a new business with zero internal equity in case of default. Debt and equity contracts are not well defined if entrepreneurs participate in the contract with zero internal equity.

Entrepreneurs indexed by i are infinitely-lived and risk-neutral. They maximize expected lifetime utility by optimally choosing consumption $c_{i,t}^e$ and capital $z_{i,t+1}$. They discount future consumption with a time discount factor $\xi\beta$ where $\xi \in (0,1)$. Note that entrepreneurs discount the future more than households. This assumption is necessary, since entrepreneurs will otherwise accumulate capital up to the point where self-financing is enough to cover the entire investment project; financial frictions would not play any role in this case. Entrepreneurs form internal equity in two different ways. First, they supply one unit of labor inelastically, and earn wage income. Secondly, they earn returns on the capital stock carried over from the previous period, in the form of consumption goods that can be used as an input for capital production. These two sources define the following equation for entrepreneurs' internal equity:

$$
n_{i,t} = w_t^e + z_{i,t}\left(q_t(1-\delta) + r_t\right),
$$

which clearly shows that internal equity $n_{i,t}$ is endogenously determined by entrepreneurs' capital accumulation decisions on $z_{i,t}$ in the previous period, in contrast to the partial equilibrium analysis where entrepreneurs take $n_{i,t}$ as given and fixed. Given internal equity, entrepreneurs tap financial institutions to finance the remaining costs of their investment project $i_{i,t}$ with debt and equity. Since the uncertainty shock is realized at the beginning of period t, the realization of the dispersion of idiosyncratic productivity $\sigma_{\omega,t}$ is common knowledge across all parties when financial contracts are made. In the presence of financial frictions in debt and equity contracts, and as long as entrepreneurs rely on external financing to some degree, the return on internal saving (or internal equity) is always greater than the return on external financing. This result, together with entrepreneurs' risk-neutrality, implies that entrepreneurs commit all of their internal equity to the project. Other details of financial contracts are identical to those in the previous section. To avoid complexity, the model abstracts from the possibility of dynamic or repeated contracts. In other words, entrepreneurs make contracts with different lenders and external shareholders each period. Once debt and equity contracts are made, the entrepreneurs' idiosyncratic productivity shocks are realized, and entrepreneurs make debt default and profit diversion decisions. Solvent entrepreneurs divide returns from their project into consumption and physical capital accumulation, which forms the basis of internal equity in the following period. In case of default, entrepreneurs consume zero units of the consumption good, and start a new business in the following period with an initial level of internal equity built by supplying a single unit of labor. Entrepreneurs that divert profits behave as solvent firms.

Formally, the utility maximization problem of solvent entrepreneurs is

$$\max_{c^e_{i,t}, z_{i,t+1}} E_0 \sum_{t=0}^{\infty} (\xi\beta)^t c^e_{i,t}$$

subject to

$$c^e_{i,t} + q_t z_{i,t+1} \leqq q_t i_{i,t} \tilde{A}_{i,t} \tag{7}$$

$$i_{i,t} = \frac{1}{1 - q_t (B_t + C_t)} n_{i,t} \tag{8}$$

$$n_{i,t} = w^e_t + z_{i,t} (q_t (1 - \delta) + r_t) \tag{9}$$

where $\tilde{A}_{i,t}$ is the realized fraction of output belonging to an individual entrepreneur. B_t and C_t are the expected share of output paid to lenders and external shareholders, respectively. Solving the maximization problem yields the entrepreneurs' intertemporal Euler equation:

$$q_t = E_t \left[\xi\beta \left\{ (q_{t+1}(1-\delta) + r_{t+1}) \frac{q_{t+1}A_{t+1}}{1 - q_{t+1}\left(B_{t+1} + C_{t+1}\right)} \right\} \right]. \tag{10}$$

As discussed in the previous section, $\frac{q_{t+1}A_{t+1}}{1-q_{t+1}(B_{t+1}+C_{t+1})}$ is the expected return on internal saving. As long as financial frictions are present and entrepreneurs finance externally, this term is greater than one. Comparing equation (6) and (10), it is clear that entrepreneurs have a stronger incentive to accumulate physical capital (which, in turn, will become next period's internal equity) than households for a given discount rate. If entrepreneurs have the same discount factor as households, they will eventually accumulate enough physical capital that they can finance investment solely with internal equity. To avoid this outcome, I assume that entrepreneurs have an additional discount factor ξ. Another important observation is that the level of internal equity does not affect the above Euler equation, which is a direct consequence of the linearity assumption; entrepreneurs have access to a CRS technology, and the monitoring and diversion costs are linear in the size of the project $i_{i,t}$. The entrepreneur subscript i does not appear in the entrepreneurs' Euler equation (recall that contract terms $\bar{\omega}$ and s are identical across entrepreneurs regardless of the level of internal equity, as shown in section 3.3). This allows me to analyze the aggregate economy without tracking the distribution of individual entrepreneurs' internal equity, which reduces the computational burden substantially. Lastly, note that $\tilde{A}_{i,t}$ in equation (7) denotes the *realized* fraction of output belonging to an individual entrepreneur. However, it is not necessary to track $\tilde{A}_{i,t}$ of each individual entrepreneur, since aggregation of entrepreneurs' budget constraint across individuals (along with equation (8) and (9)) yields the following aggregate entrepreneur budget constraint:

$$c_t^e + q_t z_{t+1} = q_t i_t A_t, \tag{11}$$

where $\tilde{A}_{i,t}$ in equation (7) is replaced by A_t, which by the law of large numbers is the expected share of output paid to the entrepreneur, defined as in Section 3.3. Note that there is no entrepreneur subscript i in equation (11) due to aggregation. It is possible to further drop subscript i in equations (8) and (9) after aggregation across entrepreneurs, and to track only aggregate variables.[10] Since the main interest is in the average behavior of agents and aggregate fluctuations, I abstract from subscript i from now on.

The role of financial institutions in this model is to channel funds (in consumption goods) from households to entrepreneurs, and to relay capital goods from entrepreneurs to households who want to purchase capital goods (recall that entrepreneurs repay in capital goods to financial institutions). In addition, financial institutions pay households a non-stochastic return from investing in equity and debt, by pooling funds from each household. Since financial institutions invest in the debt and equity of an infinite number of entrepreneurs, they can effectively diversify the risk arising from idiosyncratic productivity shocks. In particular, since there is no aggregate shock realized for the duration of the contract, households receive a gross return of 1 in terms of consumption goods from financial institutions each period regardless of the realized value of uncertainty shocks. However, the time-varying dispersion of idiosyncratic productivity shocks is an *aggregate risk*, and generates business cycles. Uncertainty shocks are aggregate shocks since they affect the terms of debt and equity contracts that *all* entrepreneurs face. As a result uncertainty shocks *do generate* aggregate fluctuations regardless of diversification, as the level of debt and equity financing of all entrepreneurs is affected by changes in the dispersion of idiosyncratic productivity shocks, although households still receive a non-stochastic return.

Market clearing conditions for the two labor markets, the consumption goods market, and

[10] To be more precise, one can compute averages by summing across entrepreneurs, then dividing by entrepreneurial mass η.

the capital goods market are given by

$$
\begin{aligned}
H_t &= (1-\eta)h_t \\
H_t^e &= \eta \\
(1-\eta)c_t + \eta c_t^e + \eta i_t &= \theta_t F(K_t,\, H_t,\, H_t^e) \\
K_{t+1} &= (1-\delta)K_t + \eta i_t \left[1 - \Phi(\bar{\omega})\mu - (1 - \Phi(\widehat{\omega}))\,\gamma\right]
\end{aligned}
$$

where $K_{t+1} = (1-\eta)k_{t+1} + \eta z_{t+1}$ and $\widehat{\omega} = \bar{\omega} + \frac{\gamma}{\phi s_t}$.

Lastly, I specify laws of motion for aggregate shocks as follows:

$$
log(\sigma_{\omega,t}) = (1-\rho_{\sigma_\omega})log(\sigma_{\omega,ss}) + \rho_{\sigma_\omega} log(\sigma_{\omega,t-1}) + \epsilon_{\sigma_\omega,t} \tag{12}
$$

$$
log(\theta_t) = \rho_\theta log(\theta_t) + \epsilon_{\theta,t} \tag{13}
$$

where $\epsilon_{\sigma_\omega} \sim \left(0, \sigma_{\sigma_\omega}^2\right)$ and $\epsilon_{\sigma_\omega} \sim \left(0, \sigma_\theta^2\right)$.

4.2 Definition of Competitive Equilibrium of DSGE Model

A competitive equilibrium is a vector of variables:

$\left\{\bar{\omega}_t,\, s_t,\, i_t,\, e_t,\, c_t^h,\, h_t,\, k_{t+1},\, c_t^e,\, z_{t+1},\, n_t,\, w_t,\, w_t^e,\, r_t,\, q_t,\, H_t^e,\, H_t,\, \theta_t,\, \sigma_{\omega,t}\right\}$ which satisfies

- Household utility maximization:

$$
\begin{aligned}
w_t &= \frac{u_L(t)}{u_c(t)} \\
q_t u_c(t) &= \beta E\left[u_c(t+1)\left\{r_{t+1} + q_{t+1}(1-\delta)\right\}\right]
\end{aligned}
$$

- Enterpreneur utility maximization:

$$
\begin{aligned}
q_t &= E_t\left[\xi\beta\left\{(q_{t+1}(1-\delta) + r_{t+1})\frac{q_{t+1}A_{t+1}}{1 - q_{t+1}\left(B_{t+1} + C_{t+1}\right)}\right\}\right] \\
c_t^e + q_t z_{t+1} &= q_t A_t i_t \\
n_t &= w_t^e + z_t\left(q_t\left(1-\delta\right) + r_t\right)
\end{aligned}
$$

- Optimal financial contract:

$$\begin{aligned}
\frac{A_{1,t}}{A_{2,t}} &= \frac{B_{1,t}+C_{1,t}}{B_{2,t}+C_{2,t}} \\
q_t &= \frac{1}{(B_t+C_t)-\frac{(B_{1,t}+C_{1,t})}{A_{1,t}}A_t} \\
i_t &= \frac{1}{1-q_t\left(B_t+C_t\right)}n_t \\
e_t &= \frac{q_tC_t}{1-q_t\left(B_t+C_t\right)}n_t
\end{aligned}$$

- Factor price:

$$\begin{aligned}
r_t &= F_K(K_t,\, H_t,\, H_t^e) \\
w_t &= F_H(K_t,\, H_t,\, H_t^e) \\
w_t^e &= F_{H^e}(K_t,\, H_t,\, H_t^e)
\end{aligned}$$

- Market clearing conditions:

$$\begin{aligned}
H_t &= (1-\eta)h_t \\
H_t^e &= \eta \\
(1-\eta)c_t+\eta c_t^e+\eta i_t &= \theta_t F(K_t,\, H_t,\, H_t^e) \\
K_{t+1} &= (1-\delta)K_t+\eta i_t\left[1-\Phi(\bar{\omega})\mu-\left(1-\Phi\left(\tilde{\omega}\right)\right)\gamma\right]
\end{aligned}$$

- Laws of motion of aggregate shocks:

$$\begin{aligned}
log(\sigma_{\omega,t}) &= (1-\rho_{\sigma_\omega})log(\sigma_{\omega,ss})+\rho_{\sigma_\omega}log(\sigma_{\omega,t-1})+\epsilon_{\sigma_\omega,t} \\
log(\theta_t) &= \rho_\theta log(\theta_t)+\epsilon_{\theta,t}
\end{aligned}$$

where $K_{t+1} = (1-\eta)k_{t+1}+\eta z_{t+1}$, and $\{\epsilon_{\sigma_\omega,t}\,\epsilon_{\theta,t}\}$ are a vector of exogenous shocks, and $\{A_t,\, B_t,\, C_t,\, A_{1t},\, B_{1t},\, C_{1t},\, A_{2t},\, B_{2t},\, C_{2t}\}$ is defined identically as in section 3.3.

4.3 Calibration

I calibrate the model at a quarterly frequency. The calibration strategy is designed to ensure comparability between the debt-equity model and the debt-only model. To achieve this goal, I closely follow the calibration strategy of Carlstrom and Fuerst (1997) and Chugh (2016). The former is used as a benchmark debt-only model, and the latter investigates the transmission of uncertainty shocks under an almost-identical setting as in Carlstrom and Fuerst (1997). Table 1 summarizes calibrated parameter values.

For both the debt-equity model and debt-only model, the household discount factor is set to $\beta = 0.99$, which is standard. Household preference is $u(c, 1-h) = ln(c) + \nu(1-h)$, where I calibrate ν so that households' labor supply h_{ss} is equal to 0.3 in the steady state.

For both models, the production function $F(K_t, H_t, H_t^e) = K_t^{\alpha} H_t^{\kappa} H_t^{e\,(1-\alpha-\kappa)}$. I set $\alpha = 0.36$, $\kappa = 0.6399$ and the capital depreciation rate $\delta = 0.02$, following standard RBC models.

A careful calibration of parameters characterizing debt and equity contracts is absolutely crucial to making a reasonable comparison between the debt-only and debt-equity models, since the role of financial frictions embedded in debt and equity contracts in the transmission of uncertainty shocks is our main interest. For the monitoring cost in case of default, I set $\mu = 0.25$ in both models. Although the cost of bankruptcy is directly observable to some degree, different studies report different values for μ. For example Altman (1984) documents that the costs of default, such as legal costs and lost sales and profits, are approximately 20 percent of total assets, while Alderson and Betker (1995) report costs of approximately 36 percent of total assets. However, the focus of this research is not on precisely estimating the cost of bankruptcy, but instead, on *comparing* two models. For the reason, I simply use the value assumed in Carlstrom and Fuerst (1997).

The calibration of remaining parameters $\sigma_{\omega,ss}$, ξ, γ, and ϕ is more complicated. For both models, the long-run average uncertainty is set to $\sigma_{\omega,ss} = 0.2466$, which I estimate from firm-level data as described in Section 6.1. I calibrate ξ of the debt-only model targeting a 240 basis points annualized real cost of borrowing in steady state ($q(1 + r_d) - 1 = 60bps$). The target moment is the average of the Baa-Treasury spread for the sample period 1993-

2014.[11] Under this parameterization, the debt-only model implies steady state quarterly default rate $\Phi(\bar{\omega}_{ss}) = 1.12\%$, which is slightly higher than the rate from Dun & Bradstreet data (quarterly default rate of 0.97%) cited by Carlstrom and Fuerst (1997). I use this value to calibrate the equity financing friction parameters of the debt-equity model.

For the debt-equity model, there are three remaining financial contract parameters, ξ, γ, and ϕ. I calibrate the parameters jointly targeting a 1.12% quarterly default rate (which is implied by the debt-only model), a 240 basis point annualized real borrowing cost on the debt contract, and an entrepreneurial share $(1 - s)$ of 0.26 in the steady state. The target moment for the steady state entrepreneurial share mostly follows Holderness et al. (1999). They document that the average managerial stock ownership of top 40th to 50th percentile firms by size is 16.2% in 1935 and 24.4% in 1995. I select a 26% share for 2005, the midpoint of the sample period 1993-2014, by linear extrapolation of the long-term trend implied by Holderness et al. (1999).[12]

For both models, I am able to exactly match all targeted moments. Under the parameterization reported in Table 1, the two models have exactly identical real costs of borrowing and default rates on debt in the steady state, implying that I can investigate the role of equity financing frictions in the transmission of uncertainty shocks by comparing the two models.

Parameter values characterizing the laws of motion of aggregate shocks (at a quarterly frequency) are calibrated to match annual persistence parameters and the variance-covariance matrix for TFP and uncertainty, as estimated from data. I calculate aggregate TFP using annual data on labor and capital inputs. Using aggregate TFP and the benchmark measure of uncertainty constructed in Section 6.1, I estimate persistence parameters and the variance-covariance matrix of uncertainty and TFP shocks at an annual frequency.[13] Then, I calibrate quarterly persistence parameters, and the variance-covariance matrix, using a simulated method of moments. In particular, I generate simulated series of uncertainty

[11] The data can be downloaded from the Federal Reserve Economic Data. In particular, the name of the series is "Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity" and the corresponding series ID is "BAA10Y".

[12] Leverage is not a targeted moment. The steady state leverage ratio is 0.565 and 0.560 for the debt-only model and the debt-equity model respectively. The corresponding data moment for leverage, defined as total assets over total liabilities, is 0.468. When defined as total debt over total assets, the data moment is 0.203.

[13] The data availability allows me to measure uncertainty only at an annual frequency. See Section 6.1 for details.

and TFP shocks for 400,000 quarters, convert simulated series into annual frequency, estimate annual persistence parameters and the variance-covariance matrix using simulated series converted to an annual frequency, and match annual persistence parameters and the variance covariance matrix from data. This procedure yields a quarterly persistence parameter of uncertainty shocks (ρ_{σ_ω}) of 0.83. This estimate is similar to Chugh (2016) who reports a persistence parameter of 0.83, estimated using data from the Longitudinal Research Database. My persistence is lower than Christiano et al. (2014) who estimate a persistence of 0.95. I estimate a standard deviation of uncertainty shocks $\sigma_{\sigma_\omega} = 0.0469$. This result is in line with previous empirical findings which document that σ_{σ_ω} ranges from 0.0374 to 0.07 (Christiano et al., 2014; Chugh, 2016). The quarterly persistence parameter of TFP shocks (ρ_θ) is 0.78. This estimate is slightly lower than the value reported in the large-scale DSGE model estimation literature (Christiano et al., 2014). The standard deviation of TPF shocks is $\sigma_\theta = 0.0071$, similar to standard RBC literature (King and Rebelo, 1999). The correlation between uncertainty and TFP shocks ($corr\ (\epsilon_{\sigma_\omega,t}, \epsilon_{\theta,t})$) is -0.5887. This is consistent with the notion that uncertainty increases during recessions, as I document further in Section 6.1.

5 Numerical Results

5.1 Impulse Response Analysis

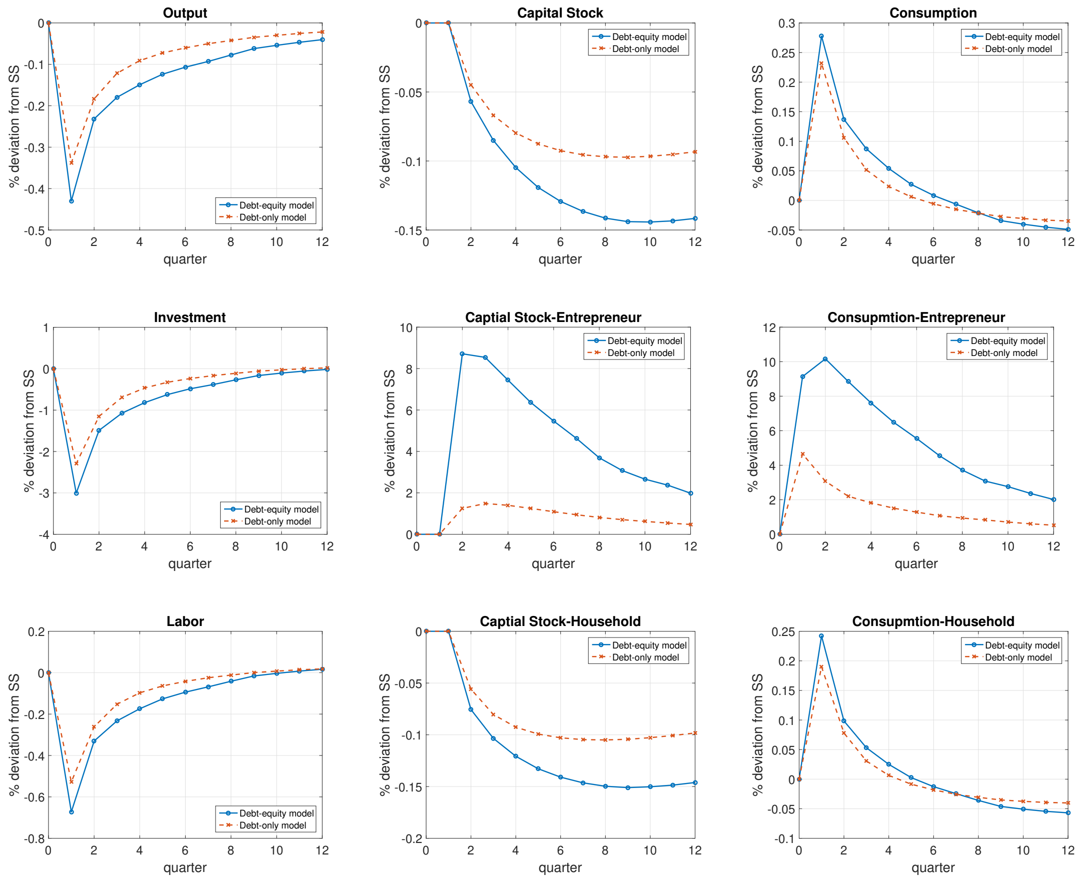
There are two goals of the impulse response analysis. The first is to understand how uncertainty shocks affect firms' financing decisions, especially equity financing decisions. The second is to understand how the response of macroeconomic variables to uncertainty shocks is *amplified* through equity financing frictions. In particular, I compare impulse response functions to uncertainty shocks of the debt-equity model with those from the debt-only model to highlight the amplification mechanism that is unique to the debt-equity model.

5.1.1 Dynamic Effect of Uncertainty Shocks

I numerically solve the model defined in Section 4.2 with a 3rd-order approximation around the deterministic steady state. The impulse I consider in this section is a one-time one-standard-deviation increase in the dispersion of idiosyncratic productivity, holding the level of aggregate TFP at its steady state. In other words, I do not take the correlation between the two shocks into account in calculating impulse response functions. Under the current calibration, this equals roughly a 4.7% increase in σ_ω, from 0.246 to 0.257. Figures 2 to 4 present impulse response functions of main variables of interest to the uncertainty shock for the debt-equity model (solid lines with circles), and the debt-only model (dashed lines with 'x'). Starting with balance sheet variables, on impact, entrepreneurs in the debt-equity model immediately downsize debt financing to approximately 4% below the steady state level (top-right panel of Figure 3). As uncertainty increases, downside risk increases, which results in an increased default probability (3rd-row left-column of Figure 4). As a result, debt financing becomes more expensive, and the real borrowing cost increases (2nd-row left-column of Figure 4) which induces entrepreneurs to lower debt financing compared to the steady state. This transmission channel of uncertainty shocks through debt financing frictions has already been discussed widely in the previous literature (Christiano et al., 2014; Gilchrist et al., 2014; Chugh, 2016), and the debt-equity model is consistent with previous findings.

However, the debt-equity model also has unique implications coming from equity financing frictions. As uncertainty increases, entrepreneurs raise less external equity than in the steady state. As discussed in the partial equilibrium analysis, increased uncertainty makes investing in external equity less attractive for two reasons. First, an increased downside risk implies that capital good producing firms are more likely to default, and thus less likely to generate positive profit. Secondly, increased upside risk increases entrepreneurs' temptation to divert profit (3rd-row right-column of Figure 4). As a result, equity financing becomes more costly to entrepreneurs, as the amount of equity they raise by selling a unit of shares s, or equivalently the stock price e/s, decreases (2nd-row right-column of Figure 4), which discourages entrepreneurs from raising external equity (middle-right of Figure 3).

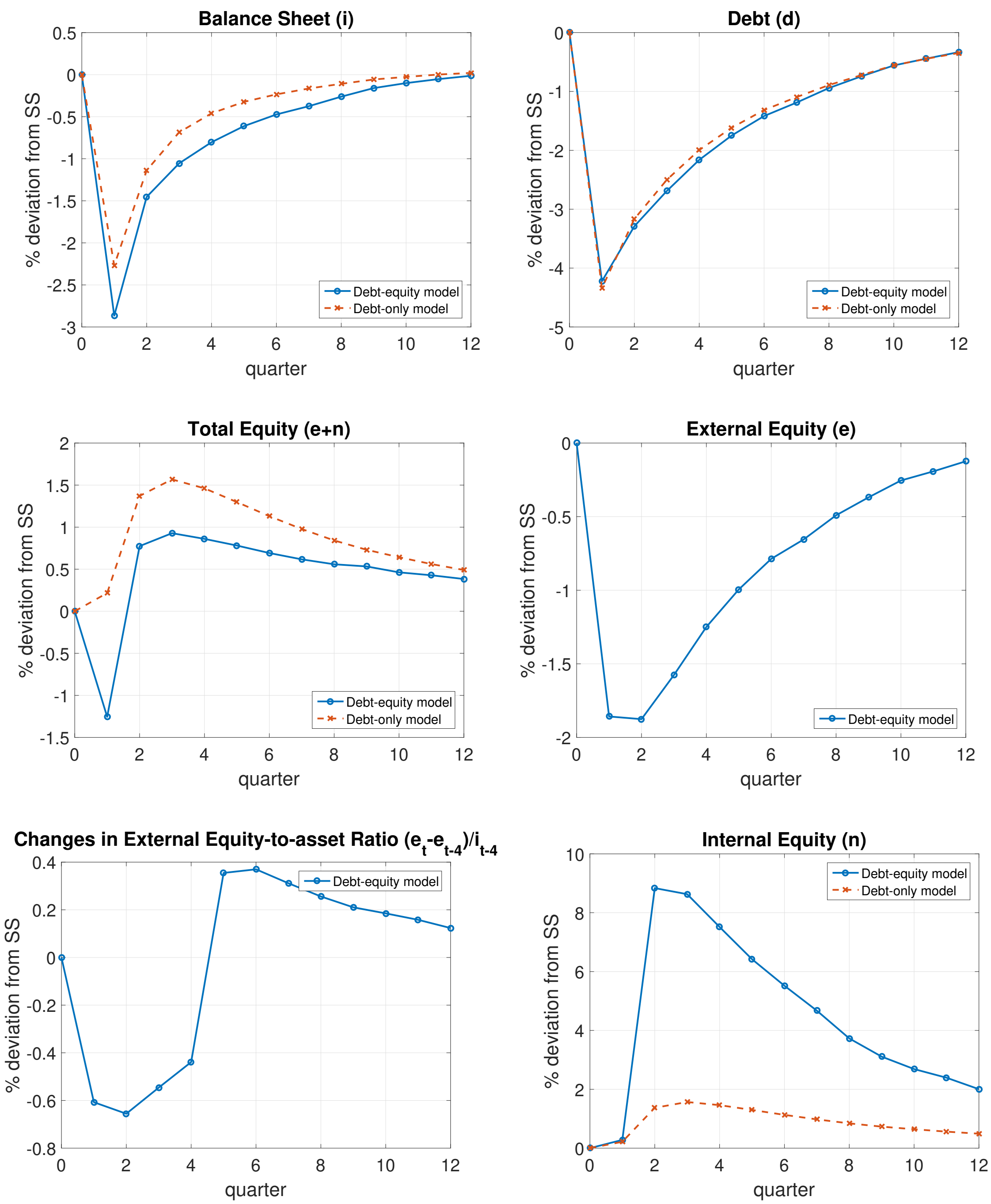
Figure 2: IRF in response to Uncertainty Shock - Aggregate variables



Notes: The figure shows impulse response functions of aggregate variables to a one-time one-standard-deviation increase in uncertainty shocks. Solid lines are from the debt-equity model. Dashed lines are from the debt-only model.

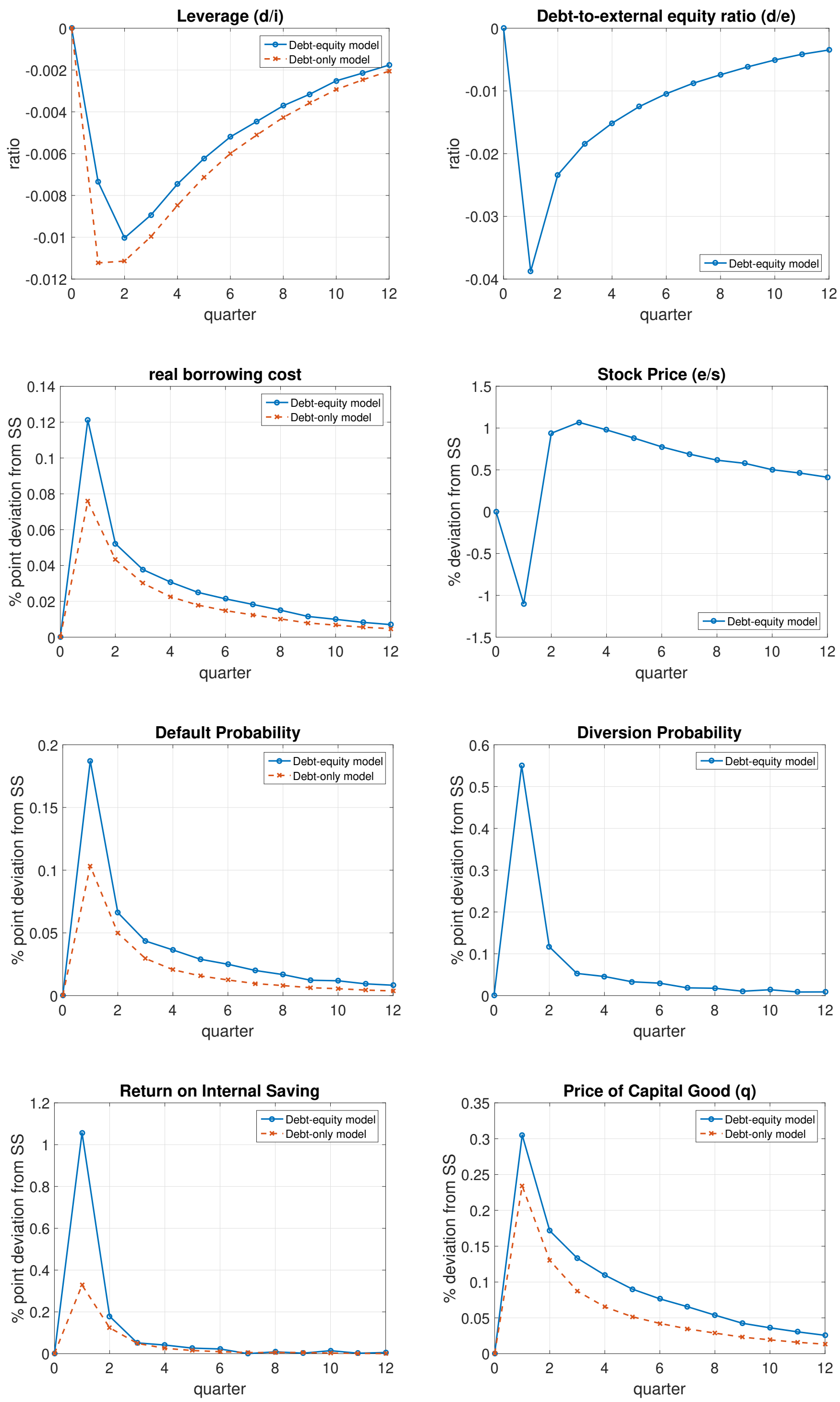
The last remaining item of the liability side of the corporate balance sheet is internal equity. Internal equity becomes more valuable as external financing becomes more costly. Clearly, increased uncertainty makes both debt and equity financing more costly. As a result, entrepreneurs are strongly motivated to accumulate internal capital (bottom-right of Figure 3 and 4th-row left-column of Figure 4). Note that entrepreneurs in the debt-equity model have a stronger incentive to build internal equity than those in the debt-only model. While there is a single source of agency costs in the debt-only model, the debt-equity model has an additional source of agency costs, namely equity financing frictions. As a result, internal saving provides a higher return in the debt-equity model than in the debt-only model in

Figure 3: IRF in response to Uncertainty Shock - Balance Sheet



Notes: The figure shows impulse response functions of balance sheet variables to a one-time one-standard-deviation increase in uncertainty shocks. Solid lines are from the debt-equity model. Dashed lines are from the debt-only model.

Figure 4: IRF in response to Uncertainty Shock - Others financial variables



Notes: The figure shows impulse response functions of other financial variables to a one-time one-standard-deviation increase in uncertainty shocks. Solid lines are from the debt-equity model. Dashed lines are from the debt-only model.

response to adverse uncertainty shocks (4th-row left column of Figure 4).

Total equity shrinks immediately and sharply on impact in the debt-only model as external equity shrinks, and overshoots above the steady state in following periods as entrepreneurs start to build internal equity to overcome the higher cost of external financing (middle-left of Figure 3). Overall, the size of the balance sheet shrinks persistently, since downward pressure from debt and external equity is greater than the upward pressure from internal equity (Top-left of Figure 3).

Comparing the impulse responses of balance sheet variables from the two models, it is clear that the effect of uncertainty shocks is amplified in the debt-equity model. I decompose the amplification effect arising from the equity contract into two components: the *direct* and *indirect* effect. As discussed above, decreasing external equity financing directly affects the size of the balance sheet. Total equity falls on impact and does not increase as much in subsequent periods compared to the debt-only model, since a persistent decrease in external equity offsets the increase in internal equity.

The indirect effect comes from the fact that the level of total equity determines the debt capacity. As total equity decreases on impact, entrepreneurs' debt capacity shrinks. As a result, debt financing is further limited when external equity decreases. In subsequent periods, total equity does not increase as much compared to the debt-only model. This creates an additional downward pressure on debt financing compared to the debt-only model. However, the impulse response functions of debt suggest that the indirect effect is small quantitatively.

Next, I discuss fluctuations of aggregate variables. Since capital good producing firms are subject to financial frictions, uncertainty shocks affect the aggregate economy mostly through the investment channel. Since the size of the balance sheet shrinks, investment falls immediately on impact (middle-left panel of Figure 2). Since the supply of capital goods decreases, the stock of capital and aggregate output decreases in response to uncertainty shocks. Again, the effect of uncertainty shocks is amplified in the debt-equity model since the balance sheet of capital producers shrinks more in the debt-equity model.

One potentially counter-factual prediction of the model is the impulse response function of consumption (3rd column of Figure 2). Aggregate consumption is counter-cyclical in response to uncertainty shocks. As discussed in Barro and King (1984) and Chugh (2016), standard RBC models do not predict a procyclical impulse response function of consumption to shocks that do not affect the marginal productivity of labor or labor supply directly. The uncertainty shock falls into that category.

Since the model is solved with a 3rd-order approximation, I can examine whether there are non-linear effects of uncertainty shocks. First, I compare impulse response functions to a one-time one-standard-deviation increase and decrease in uncertainty (hereafter a 1-SD adverse and favorable shock, respectively) generated by the debt-equity model. Figure 5 reports impulse response functions of key variables to these two shocks. The peaks of the main financial variables (cost of borrowing, default probability, and diversion probability) generated by a 1-SD adverse shock are slightly larger than the troughs generated by a 1-SD favorable shock. The outcomes of financial contracts affect firms' investment decision directly, and as a consequence the trough of investment in response to a 1-SD adverse shocks is slightly larger than the peak. However, regardless of these slight differences, the shapes of the impulse response functions to these two shocks are largely symmetric implying, that asymmetries in the effects of uncertainty shocks are only of second order importance.

I also compare impulse response functions to one-standard-deviation and two-standard-deviation adverse uncertainty shocks generated by the debt-equity model. Results reported in Figure 6 non-linearities are virtually nonexistent; the impulse response functions to a two standard deviation shock are almost identical to the impulse response functions to a one standard deviation shocks scaled by a factor of two.

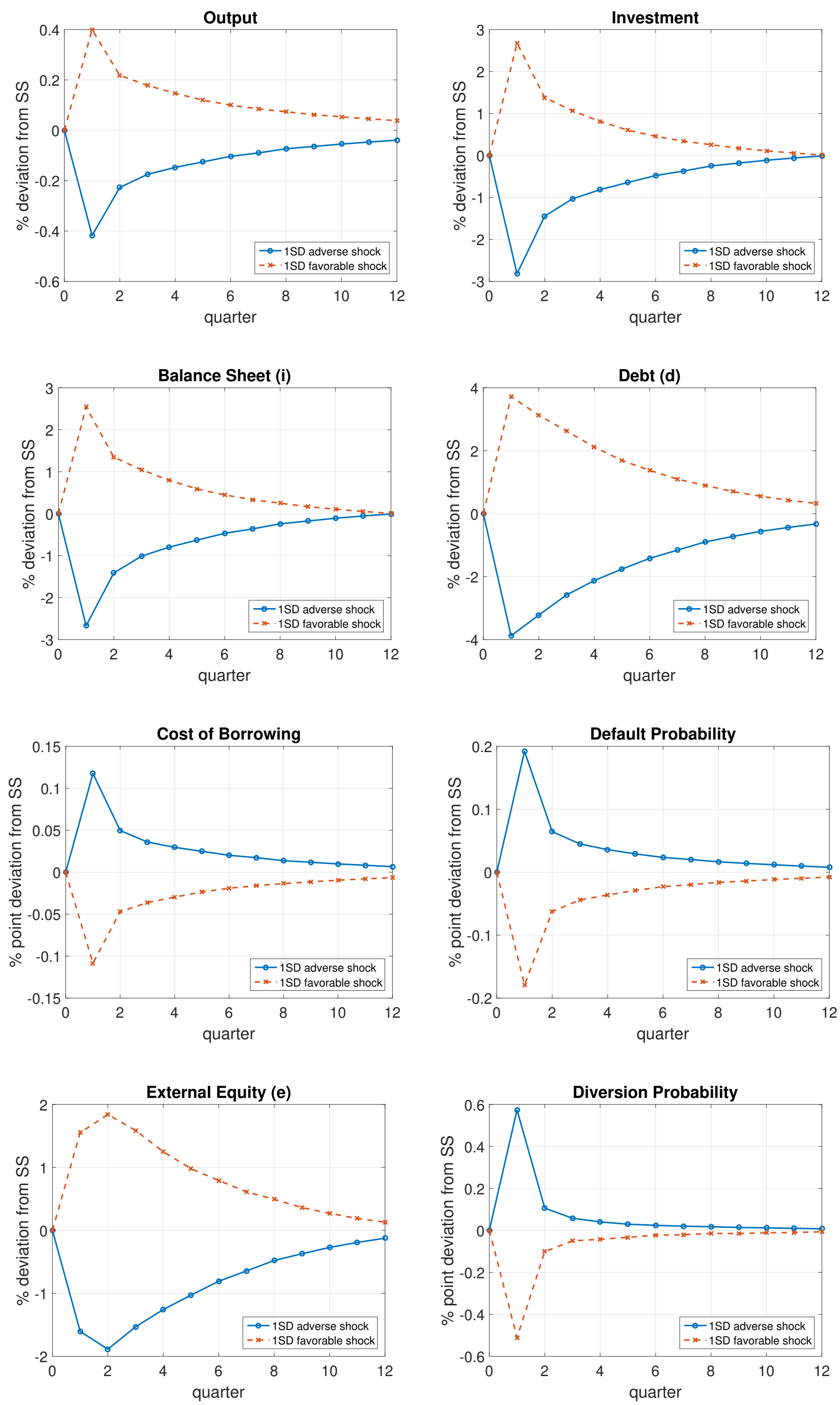
Financial frictions in debt and equity contracts are the core channel of the transmission mechanism in my model. The two contractual parties, firms and financial institutions, are assumed to be risk neutral. As a result, the effects of uncertainty shocks are captured mostly by linear terms, while non-linear effects of uncertainty shocks remain limited. However, I do not conclude that non-linear effects would not matter in a richer model. The present model is simplified to highlight the role of equity financing in amplification of uncertainty shocks. Studying non-linear effects requires a richer model which is beyond the scope of this research.

5.1.2 Dynamic Effect of TFP Shocks

In this section, I investigate how the model responds to a one-time one standard deviation decrease in TFP, holding uncertainty fixed at its steady state level. This is approximately an 0.7% drop in TFP from the steady state. Impulse responses to the TFP shock are shown in Figures 7 to 9. As reported in Figure 8, both debt and external equity financing decrease in response to negative TFP shocks (top-right and middle-right panel of Figure 8). However, the underlying reason for decreasing external financing is different from the case of uncertainty shocks. Since the marginal product of capital decreases in response to decreased TFP, households demand less capital goods. As a result, entrepreneurs reduce the size of the balance sheet not because of tighter financial constraints, but simply to meet a reduced demand for capital goods. This is also reflected in a decrease in the price of capital (4th-row right-column of Figure 9). Entrepreneurs reduce debt and external equity financing for any given level of internal equity. In other words, the demand for credit shifts in. As a result, both the default probability and the diversion probability decrease (3rd-row left-column and 3rd-row right-column of Figure 9), which leads to a decreasing real borrowing cost (2nd-row left-column in Figure 9). Internal equity shrinks too (bottom-right and middle-right panel of Figure 8), since agency costs decrease, which implies a decrease in the return on internal saving (4th-row left-column of Figure 9).

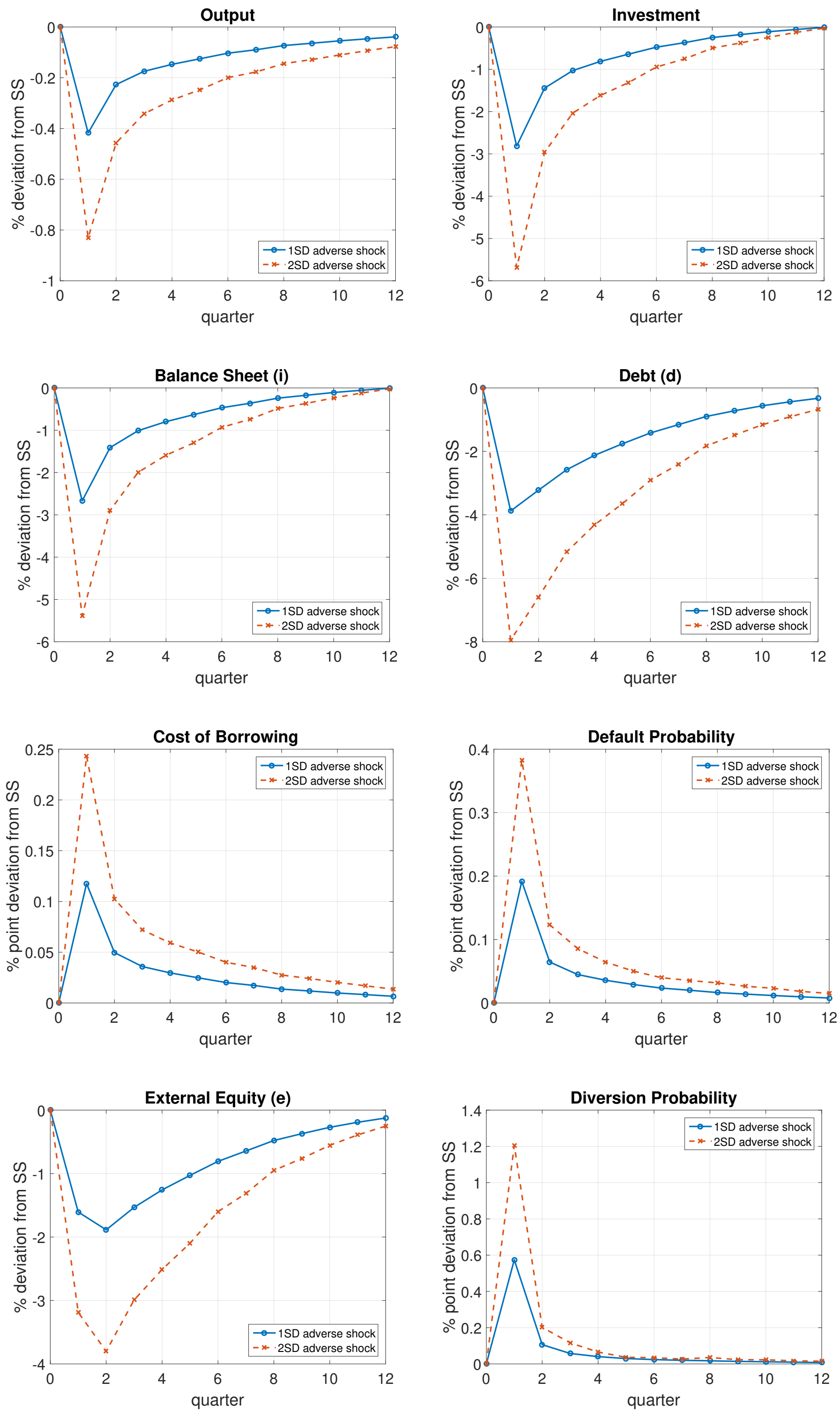
The debt-equity model predicts a larger amplification of macroeconomic variables in response to TFP shocks compared to the debt-only model. However, the amplification is concentrated at the early stage of the dynamics. In response to negative productivity shocks, entrepreneurs downsize their balance sheet in response to the reduced demand for capital goods. In the debt-equity model, entrepreneurs have two margins of *active* adjustment, debt and external equity, while in the debt-only model, entrepreneurs only have a single margin of adjustment. Recall from equations (3) and (4) that the levels of debt and external equity financing are linear functions of the level of internal equity. Thus, the size of the balance sheet shrinks at a faster rate in the debt-equity model per unit decrease in internal equity.

Figure 5: IRF in Response to Adverse and Favorable Uncertainty Shock



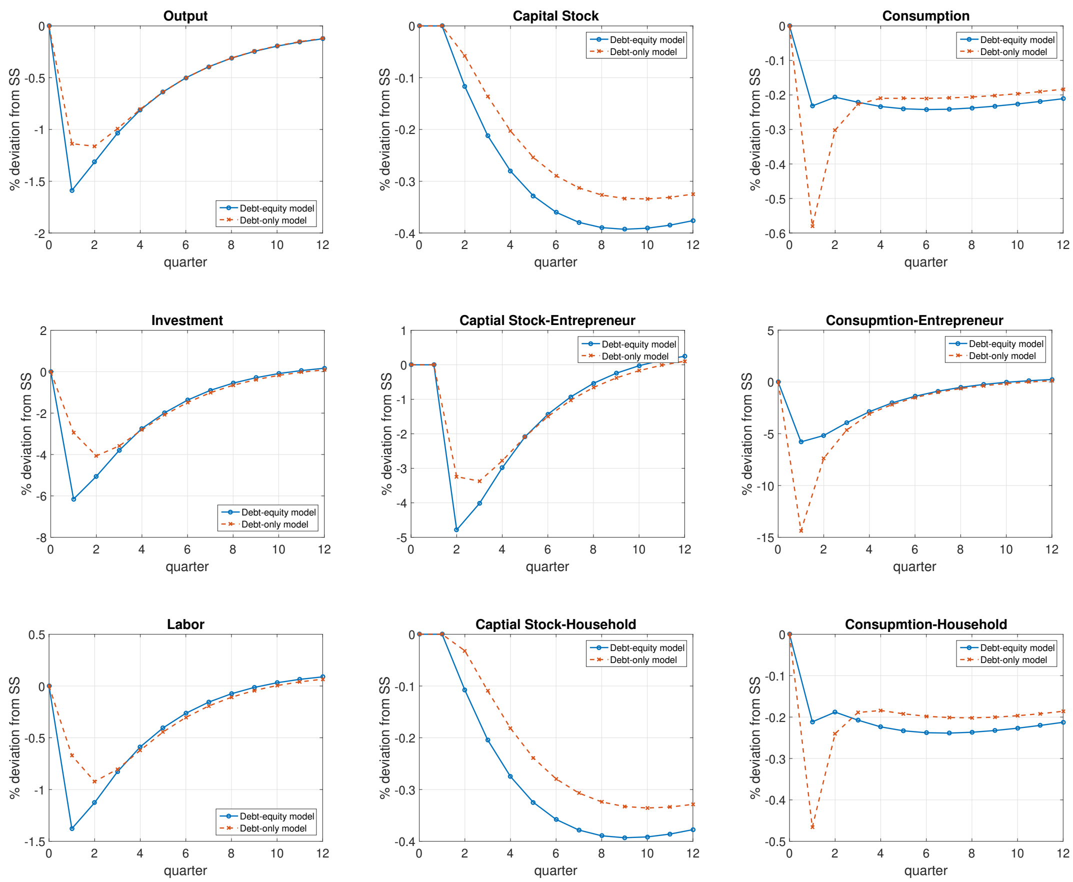
Notes: The figure shows impulse response functions to one-time one-standard-deviation increases and decreases in uncertainty generated by the debt-equity model. Solid lines are IRFs to adverse uncertainty shocks and dotted lines are IRFs to favorable uncertainty shocks.

Figure 6: IRF in Response to 1-SD and 2-SD Uncertainty Shock



Notes: The figure shows impulse response functions to one-time one-standard-deviation and two-standard-deviation increases in uncertainty generated by the debt-equity model. Solid lines are IRFs to 1-SD uncertainty shocks and dotted lines are IRFs to 2-SD uncertainty shocks.

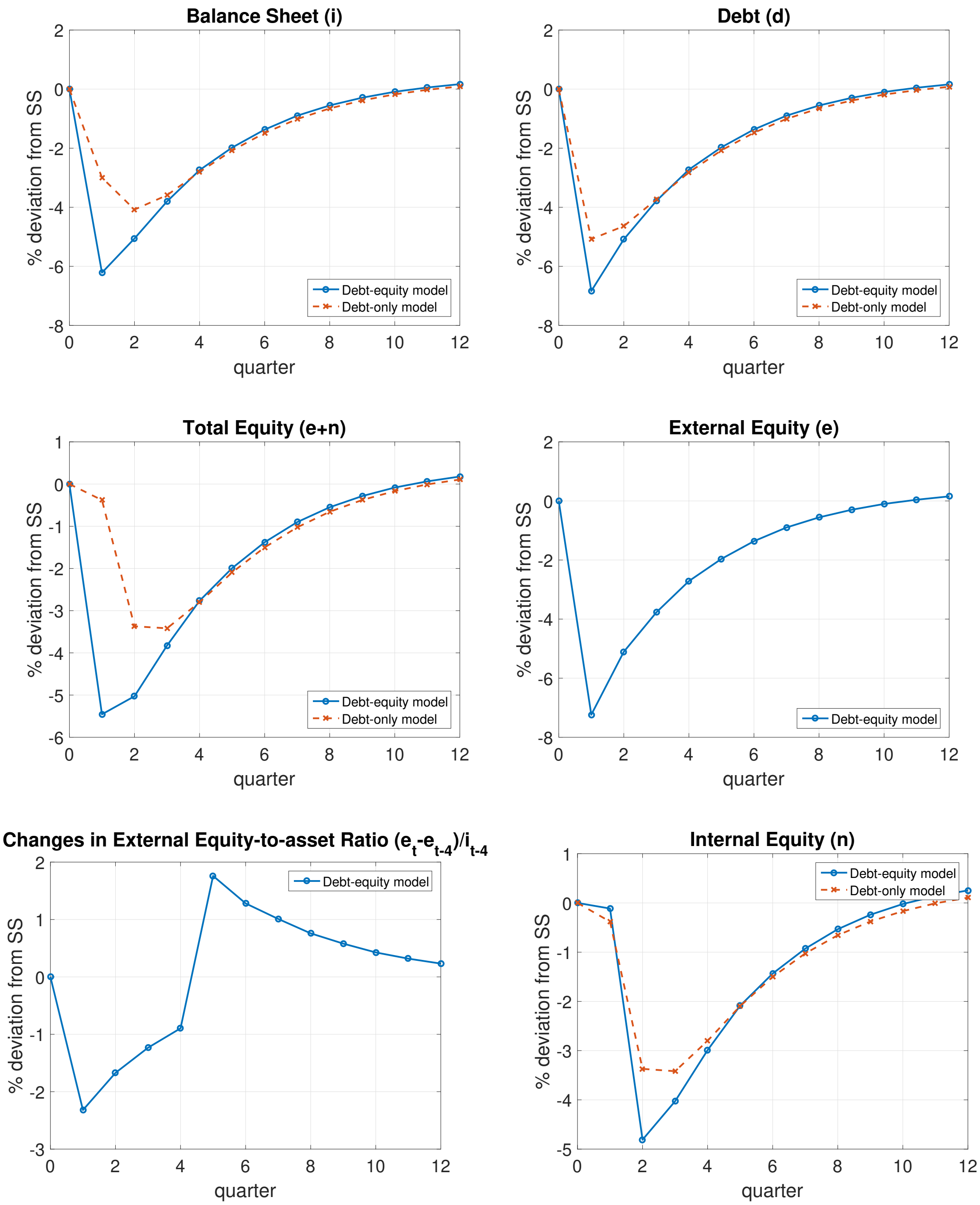
Figure 7: IRF in response to Productivity Shock - Aggregate variables



Notes: The figure shows impulse response functions of aggregate variables to a one-time one-standard-deviation decrease in TFP. Solid lines are from the debt-equity model. Dashed lines are from the debt-only model.

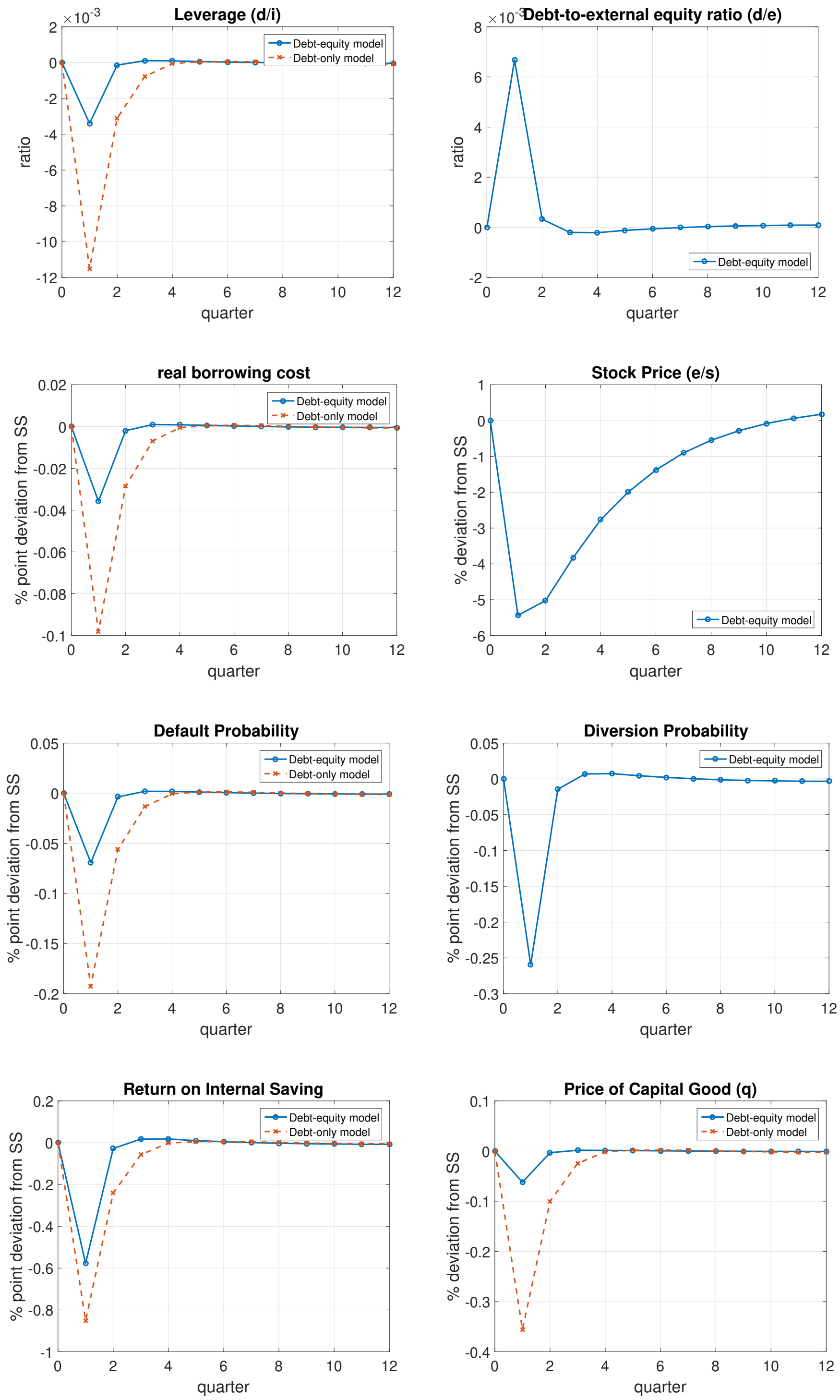
Aggregate variables are all positively correlated with TFP shocks as expected. Output falls mostly because of the decrease in TFP and the capital stock. The decrease in investment and the capital stock is a direct consequence of the decrease in the size of the balance sheet. Impulse response functions of aggregate variables to TFP shocks are amplified in the debt-equity model relative to the debt-only model. This is a direct consequence of the different dynamics of the size of the balance sheet. In contrast to the case of uncertainty shocks, consumption decreases as well.

Figure 8: IRF in response to Productivity Shock - Balance Sheet



Notes: The figure shows impulse response functions of balance sheet variables to a one-time one-standard-deviation decrease in TFP. Solid lines are from the debt-equity model. Dashed lines are from the debt-only model.

Figure 9: IRF in response to Productivity Shock - Other financial variables



Notes: The figure shows impulse response functions of other financial variables to a one-time one-standard-deviation decrease in TFP shocks. Solid lines are from the debt-equity model. Dashed lines are from the debt-only model.

5.2 Simulation: Countercyclical External Financing Costs

In this section, I simulate the model economy to investigate its cyclical properties. I account for the estimated variance-covariance matrix of TPF and uncertainty shocks as described in Section 4.3. Note that the correlation between two shocks is -0.5587 which is consistent with the notion that uncertainty increases during recessions. I simulate the model economy for 3,000 quarters where the initial 100 periods are dropped, which is a standard procedure. Table 2 reports the correlation coefficients of macroeconomic variables with aggregate output. The first column shows the sample correlation from the data for the sample period 1993Q1-2014Q4. Output, consumption, investment and stock price are logged. All data moments are calculated using HP-filtered series with a smoothing parameter $\lambda = 1,600$. Both consumption and investment are highly procyclical. I use the Baa-Treasury spread and the delinquency rate of industrial and commercial loans to calculate the data moments for real borrowing cost and the default rate, respectively. Both real borrowing cost and the default rate are countercyclical, which suggests that debt financing frictions worsen during recessions. I use the Russell 3000 index as my measure of the stock price, which is highly procyclical, implying that entrepreneurs can raise more equity by issuing shares during booms. In other words, equity financing is less costly during upturns.

The second and the third column of Table 2 report the model moments. The most important finding is that, consistent with the data, the debt-equity model generates a countercyclical real borrowing cost and default rate while the debt-only model generates the opposite. The additional amplification of uncertainty shocks due to equity financing frictions is key to explaining the difference between the two models. In both models, the real borrowing cost and the default rate decrease in response to negative TFP shocks, as discussed above. In contrast, adverse uncertainty shocks increase both the default risk and the cost of borrowing. If the effect of adverse uncertainty shocks dominates the effect of adverse TFP shocks, then the model will generate a countercyclical real borrowing cost and default rate. It turns out that the effect of uncertainty shocks dominates in the debt-equity model, but not in the debt-only model. In the debt-equity model, entrepreneurs cannot delever as much as they do in the debt-only model since equity financing is also limited due to increased uncer-

Table 2: Simulation - Cyclicality of Macroeconomic Variables

Variable	Data	Debt-only model	Debt-equity model
Output	1.00	1.00	1.00
Consumption	0.84	0.68	0.41
Investment	0.87	0.96	0.96
Real borrowing cost, $q(1+r_d)-1$	-0.50	0.11	-0.57
Default rate, $\Phi(\bar{\omega})$	-0.76	0.37	-0.50
Stock price, e/s	0.82	-	0.87
Diversion probability, $1-\Phi(\widehat{\omega})$	-	-	-0.31

Notes: The table reports the correlation between each macroeconomic variable and aggregate output. I simulate the economy for 3000 quarters where initial the 100 periods are dropped. Output, consumption, investment, and stock price are logged before sample statistics are calculated. I use the Baa-Treasury spread (FRED series ID: BAA10Y) to calculate the data moment for the real borrowing cost. I use the delinquency rate of industrial and commercial loans (FRED series ID: DRBLACBS) to calculate the data moment for the default rate. I use the Russell 3000 index (FRED series ID: RU3000PR) to calculate the data moment for the stock price. All data moments are calculated using HP-filtered series with a smoothing parameter $\lambda = 1600$. The sample period is 1993Q1-2014Q4.

tainty (see leverage in Figure 4). This is the fundamental reason why the increase in the default rate and the real cost of borrowing is further amplified in the debt-equity model. At the same time, the debt-equity model also predicts countercyclical equity financing costs. This is shown by the countercyclical diversion probability and the procyclical stock price. Adverse uncertainty shocks play a key role, as increased uncertainty worsens agency costs arising from the equity contract.

Table 3 reports the cyclicality of balance sheet variables. Since adverse TPF and uncertainty shocks both affect both debt and equity financing negatively, financing variables are all procyclical. These findings are consistent with previous empirical studies, for example Covas and Den Haan (2011).

In sum, the simulation result shows that the debt-equity model predicts both procyclical debt and equity financing, and countercyclical default and external financing costs. In contrast, the debt-only model fails to generate a countercyclical default rate and real cost of borrowing, which reconfirms the importance of equity financing frictions that amplify the effect of uncertainty shocks. This finding is important, as existing literature is unable to explain the coexistence of procyclical debt and equity financing and countercyclical external

Table 3: Simulation - Cyclicality of Balance Sheet Variables

Variable	Debt-only model	Debt-equity model
Balance sheet, i	0.96	0.96
Debt, d	0.95	0.95
External equity, e	-	0.96
Total equity, $e+n$	0.59	0.88
Leverage, d/i	0.79	0.72

Notes: The table reports correlations between each balance sheet variables and aggregate output. I simulate the economy for 3000 quarters, where the initial 100 periods are dropped, a standard procedure. All variables are logged except leverage.

financing costs (Covas and Den Haan, 2012; Jermann and Quadrini, 2006, 2012).

Both the debt-only and the debt-equity model generate procyclical investment and consumption (Table 2). For both models, investment is highly procyclical and the cyclicality is slightly stronger than the data. In contrast, consumption is substantially less procyclical in both models compared to the data. This is mainly due to the response of consumption to adverse uncertainty shocks. Consumption increases in response to adverse uncertainty shocks on impact (top-right of Figure 2), while it decreases in response to adverse TFP shocks (top-right of Figure 7). Considering that the correlation coefficient of uncertainty and TFP shocks is negative $corr\left(\epsilon_{\sigma_\omega,t}, \epsilon_{\theta,t}\right) = -0.5887$, the decrease in consumption due to negative TFP shocks is offset on average by the increase in consumption due to adverse uncertainty shocks in many cases. As a result, the procyclicality of consumption is weaker in both models. Since uncertainty shocks are more amplified in the debt-equity model than in the debt-only model, the procyclicality of consumption is weaker in the debt-equity model.

6 Empirical Evidence

Uncertainty shocks have real consequences by affecting firms' decisions over debt and equity financing. While previous studies mostly focus on the role of debt financing as a potential transmission channel of uncertainty shocks (Gilchrist et al., 2014; Christiano et al., 2014), this paper highlights the importance of equity financing. Thus, it is important to show empirically how firms' equity financing decisions respond to changes in uncertainty.

In this section, I document the relationship between firms' equity financing decisions and the level of uncertainty and provide suggestive evidence that uncertainty and equity financing are negatively correlated. To do so, I construct a measure of uncertainty taking a bottom-up approach. I estimate firm-level revenue-based total factor productivity (TFPR) using annual balance sheet data of U.S. listed firms from Compustat, and define uncertainty as the cross-sectional standard deviation of estimated firm-level TFPR.[14] This approach directly matches the model counterpart of uncertainty, which is the dispersion of firms' idiosyncratic productivity shocks. I then document how firms' equity financing decisions are associated with the level of uncertainty by closely following the regression-based approach of Covas and Den Haan (2011), who document cyclical patterns of debt and equity financing. I close the section with various robustness tests, which all suggest a negative relationship between the level of uncertainty and equity financing.

6.1 Measuring Uncertainty

I follow Wooldridge's extension of Levinsohn and Petrin (2003) to estimate firm-level revenue-based total factor productivity. I first estimate the following production function using firm-level data for a particular industry:

$$\ln\left(VA_{i,j,t}\right) = \beta_j + \beta_j^L \ln L_{i,j,t} + \beta_j^K \ln K_{i,j,t} + \epsilon_{i,j,t} \tag{14}$$

where $VA_{i,j,t}$, $L_{i,j,t}$ and $K_{i,j,t}$ represent value-added, the number of employees, and the beginning-of-period capital stock of firm i, in industry j, at time t respectively.[15] I use annual balance sheet data of U.S. listed firms from Compustat for the sample period 1990 to 2014. Utility and financial firms are excluded from the sample. Equity financing has become an important source of external financing since the early 1980s, so it would be preferable to estimate firm-level TFPR for a sample period including the early 1980s. However, the sam-

[14] I use total factor productivity (TFP) and revenue-based total factor productivity (TFPR) interchangeably hereafter within Section 6. They both refer to TFPR.

[15] See Appendix for details of how variables are constructed.

ple period starts in 1990 due to limited data availability of the industry-level value-added deflator, the intermediate goods price deflator, and the average annual wage, which are necessary for the estimation. I use a 2-digit North American Industry Classification System (NAICS) level value-added price deflator from the Bureau of Economic Analysis to convert nominal $VA_{i,j,t}$ into real terms. 3- and 4-digit deflators are used when available. Beginning-of-period capital stocks are deflated using the aggregate non-residential fixed investment deflator. Although the Bureau of Labor Statistics provides an investment good price deflator at the 2- and 3-digit NAICS level, the series starts only in 1990. Since the stock of capital is built by summing the sequence of investment over time, it is necessary to know the price deflator of investment goods purchased prior to 1990, unless the firm's investments are all made after 1990. For this reason, there is a substantial loss of observations if an industry level investment good price deflator is used instead of the aggregate price deflator.

Although it is more common to use plant-level data to estimate factor elasticities and productivity (Olley and Pakes, 1996; Levinsohn and Petrin, 2003), I use firm-level data, since plant-level data is usually available only for the manufacturing sector, which imposes a substantial limit on the scope of analysis. By using firm-level data, it is possible to extend the scope of analysis beyond manufacturing.

Since it is common to assume that each industry has different factor elasticities of labor and capital, I allow β_j^L and β_j^K to vary across industry j, and estimate the production function separately for each industry at the 2-digit NAICS level. The estimation results for 18 industries are reported in Table 4. The estimation results seem reasonable, as the sum of labor and capital elasticities is 0.87 on average across industries.

We can further decompose total factor productivity $\epsilon_{i,j,t}$ into $e_{i,j,t}$, which is known to firms based on past information (or in other words, is a rational forecast of productivity) and $u_{i,j,t}$, which is a pure *shock* component ($\epsilon_{i,j,t} = e_{i,j,t} + u_{i,j,t}$). The model defines uncertainty as the dispersion of idiosyncratic productivity *shocks*. Hence, it is necessary to estimate $u_{i,j,t}$ using estimated $\hat{\epsilon}_{i,j,t}$. To do so, I assume that the firm-level productivity evolves following an AR(1) process:

$$\hat{\epsilon}_{i,j,t} = \rho\hat{\epsilon}_{i,j,t-1} + \eta_i + \lambda_{j,t} + u_{i,j,t}. \tag{15}$$

Table 4: Factor Elasticities

Industry (2-digit NAICS)	Num. of obs	Labor (β_j^L)	Capital (β_j^K)	$\beta_j^L + \beta_j^K$
Agriculture, Forestry, Fishing and Hunting (11)	294	0.45	0.33	0.78
Mining, Quarrying, and Oil and Gas Extraction (21)	4,583	0.34	0.49	0.83
Construction (23)	1,117	0.78	0.10	0.89
Manufacturing (31)	4,096	0.48	0.32	0.80
Manufacturing (32)	10,472	0.64	0.15	0.79
Manufacturing (33)	25,139	0.64	0.11	0.74
Wholesale Trade (42)	3,467	0.77	0.17	0.94
Retail Trade (44)	3,539	0.75	0.19	0.94
Retail Trade (45)	2,011	0.60	0.19	0.79
Transportation and Warehousing (48)	2,712	0.35	0.50	0.85
Information (51)	11,049	0.81	0.06	0.87
Professional, Scientific, and Technical Services (54)	4,326	0.93	0.11	1.04
Administrative and Support and Waste Management and Remediation Services (56)	2,110	0.75	0.17	0.91
Educational Services (61)	334	0.89	0.12	1.01
Health Care and Social Assistance (62)	1,340	0.77	0.23	0.99
Arts, Entertainment, and Recreation (71)	622	0.39	0.25	0.64
Accommodation and Food Services (72)	2,389	0.62	0.24	0.87
Other Services (except Public Administration) (81)	345	0.80	0.21	1.01

Notes: The table reports the results (factor elasticities) from estimating a production function separately for each industry at a 2-digit NAICS level. The estimation methodology follows Wooldridge's (2009) extension of Levinsohn and Petrin (2003).

Following Bloom et al. (2012) and Gopinath et al. (2015), the equation (15) includes an industry-time-fixed effect $\lambda_{j,t}$ in order to capture a time-varying industry-wide component of total factor productivity. In this research, it is particularly important to distinguish aggregate (or industry-level) and idiosyncratic components of TFP considering that the main focus of this research is to investigate the effect of changes in the within-industry dispersion of *idiosyncratic* productivity. I define the OLS residual $\hat{u}_{i,j,t}$ from estimating equation (15) as the idiosyncratic productivity shock of firm i in industry j at time t.

The TFP estimation yields a total of 68,379 firm-year TFP observations for 8,416 unique firms, which is approximately 85% of the sample used in the main regression discussed in the next section, both in terms of the number of observations and unique firms. There are fewer observations available for the variables required to construct TFP than for the main regression.[16] I do not require firms included in the main regression to have all the information required to be included in the TFP regression. I follow this strategy to maximize the number of observations for the main regression.

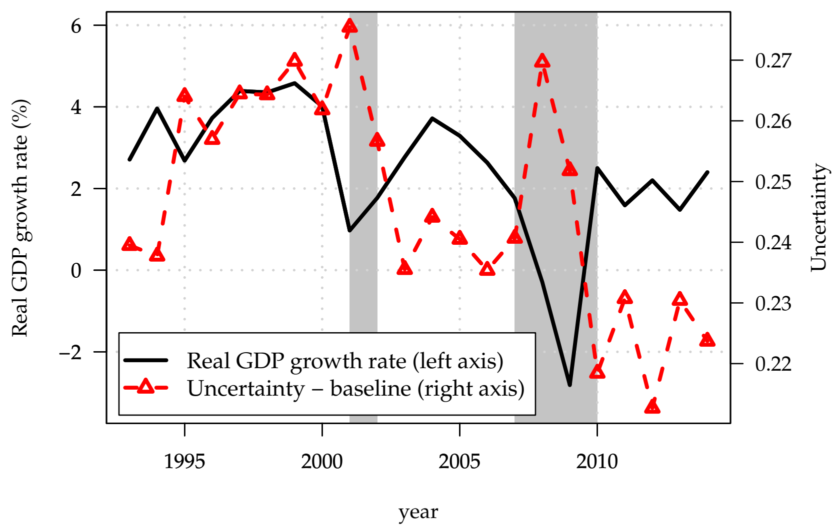
The construction of my measure of uncertainty follows naturally as the weighted average of the industry-level standard deviation of idiosyncratic productivity *shocks* at a given year t: $Uncertainty_t = \sum_j w_j \times SD_{j,t}(\widehat{u}_{i,j,t})$ where $SD_{j,t}(\widehat{u}_{i,j,t})$ is the within-industry cross-sectional standard deviation of $\widehat{u}_{i,j,t}$ at a given period t and w_j is a time-invariant weight for each industry given by the fraction of aggregate value-added accounted for by industry j. Time-invariant weights ensure that uncertainty does not vary over time due to changes in industry composition. Figure 10 shows the annual GDP growth rate and $Uncertainty_t$ for the sample period 1993-2014. $Uncertainty_t$ spikes up during the recessions in 2001 and 2008, consistent with the notion that uncertainty increases during recessions.

6.2 Uncertainty and Cyclicality of Equity Financing

Following Covas and Den Haan (2011), I take a regression approach to investigate the relationship between uncertainty and equity financing, using annual balance sheet data of

[16] Note for instance that there is an additional loss of observations from estimating equation (15) since an observation at t is dropped if an observation at $t-1$ is missing.

Figure 10: Uncertainty and Real GDP Growth



Notes: The figure shows aggregate uncertainty, $Uncertainty_t$ (solid line) and annual real GDP growth (dotted line with triangles) for the sample period 1993-2014. $Uncertainty_t$ is defined as the standard deviation of firm-level TFP shocks. See equation (14) and (15) along with the main text for the details of calculation. Annual real GDP growth is defined as the log difference of real GDP (multiplied by 100). Shaded areas show NBER recession dates. 2001-2014.
Source: Author's calculation, Compustat, Bereau of Economic Analysis

U.S. listed firms from Compustat for the sample period 1993 to 2014.[17] Utility and financial firms are excluded from the sample. If a firm-year observation violates the accounting identity that total assets equals total liabilites plus stockholders' equity by more than 10%, it is dropped from the sample in order to ensure data reliability. I describe the definition and construction of variables in more detail in Appendix. Table 5 reports sample statistics of firm characteristics for the entire sample. There are 78,149 firm-year observations from 10,595 unique firms. On average, approximately 3,500 observations (or firms) are available per year. The number of firms in the sample is similar to related previous studies using

[17]Note that I estimate firm level TFP for a sample period starting in 1990, not 1993. This is solely because of observations lost due to lagged variables. In estimating TFP, lagged values of dependent variables are required. As a result, observations in 1990 are lost. In estimating equation (15), an additional year of observations are lost. This allows me to construct $Uncertainty_t$ starting only in 1992. Lastly, I include lagged $Uncertainty$ as an additional regressor (see equation (16)) which results in an additional loss of observations in 1992.

Compustat, for example Fama and French (2005).

On average, sample firms have assets worth $3,200 million 2009 USD. The distribution of total assets is skewed to the right. Firms' average sales are approximately $2,600 million 2009 USD, and this distribution is also skewed to the right. Net stock sales are on average 14% of beginning-of-period total assets (or lagged total assets). This number is similar to Fama and French (2005), who document that average equity issues by listed firms in the U.S. in a given year during 1993-2002 represent 12.6% of total assets.

The bottom half of Table 5 reports summary statistics of firm-year observations for the subset of data with positive net stock sales. There are 43,994 firm-year observations associated with positive net stock sales, accounting for approximately 55% of the entire sample of firm-year observations. There are 9,481 unique firms, roughly 95% of all unique firms in the entire sample, that experience positive net stock sales at least once during the sample period. On average, firms report four years of positive net stock sales (43,994 firm-year observations with positive net stock sales divided by 10595 unique firms). Considering that, on average, sample firms have seven years of observations (78,149 firm-year observations divided by 10,595 unique firms), approximately half of the observations per firm are associated with positive net stock sales.

Table 5 reveals important differences between net equity issuers compared to other firms. Net equity issuers are smaller in terms of asset size and total sales, and have higher growth opportunities as measured by Tobin's Q. This is consistent with predictions of corporate finance theories, that smaller firms with high growth opportunities actively issue equity in financial markets.

The specification of the main regression equation is

$$Equity\,finance_{i,t} = \eta_i + \sum_{p=0}^{L=1} \theta_p \Delta RGDP_{t-p} + \sum_{p=0}^{L=1} \beta_p Uncertainty_{t-p} + \gamma X_{i,t} + \epsilon_{i,t} \tag{16}$$

where $Equity\,finance_{i,t}$ is the net amount of external equity raised by firm i at time t, $\Delta RGDP_t$ is the real GDP growth rate, $Uncertainty_t$ is the level of uncertainty, and $X_{i,t}$ is a vector of firm-level control variables. A firm fixed effect η_i is included to control for time-invariant

Table 5: Sample Statistics

Firm_type	Num of Obs	Num of firms	Variable	Mean	SD	Min	Median	Max
All	78149	10595	Total assets	$ 3,202.49	$ 11,519.22	$ 1.19	$ 230.68	$109,919.10
			Sales	$ 2,631.94	$ 9,061.47	$ 0.06	$ 213.79	$ 83,021.44
			Net stock sales	14.20%	51.25%	-300.00%	0.08%	300.00%
			Gross stock sales	15.80%	51.00%	0.00%	0.43%	300.00%
			Tobin's Q	3.42	8.05	0.28	1.61	85.96
			Debt-to-asset Ratio	0.20	0.19	0.00	0.17	0.77
Net Issuer	43994	9481	Total asset	$ 1,980.09	$ 8,343.25	$ 1.19	$ 171.42	$109,919.10
			Sales	$ 1,553.83	$ 6,185.23	$ 0.06	$ 146.55	$ 83,021.44
			Net stock sales	27.26%	65.18%	0.00%	1.31%	300.00%
			Gross stock sales	27.68%	65.52%	0.00%	1.48%	300.00%
			Tobin's Q	4.57	9.95	0.28	1.94	85.96
			Debt-to-asset Ratio	0.19	0.19	0.00	0.14	0.77

Notes: The table reports summary statistics of the sample. The top half of the table reports summary statistics for the entire sample, while the bottom half reports statistics for firm-years with positive net stock sales. Total assets (AT) and sales (SALE) are measured in terms of millions of 2009 constant U.S. Dollars. Net stock sales are measured as sales of common and preferred stock (SSTK) minus purchases of common and preferred stock (PRSTKC) normalized by beginning-of-year total assets, which equal lagged total assets (AT). Gross stock sales are measured as sales of common and preferred stock (SSTK) normalized by beginning-of-year total assets. Capital letters in parentheses represent Compustat variable mnemonics. Tobin's Q is measured as the book-to-market value ratio of total assets. The debt-to-asset ratio is measured as total debt divided by total assets. See Appendix for details of variable definitions. Gross and net stock sales are winsorized at 300% and -300%. All other variables are winsorized at the top and bottom 0.5 percentiles.
Source: Compustat

firm-specific factors that could affect financing decisions. I measure $Equity\,finance_{i,t}$ as net sales of stock normalized by lagged total assets as in Covas and Den Haan (2011). While $Equity\,finance_{i,t}$ can be measured in other ways, for example as changes in stockholders' equity at book or market value, net sales of stock is strictly preferred considering that this paper highlights the role of equity raised by selling stock to outside shareholders. Note that the book value of equity changes not only due to sales of stock but also due to changes in retained earnings. Considering that retained earnings are defined as profits that are not paid to stockholders, they should be considered as an internal source of financing. Meanwhile, the market value of stockholders' equity may vary for reasons other than sales of stock or changes in retained earnings. For example, if stock prices change, the market value of equity changes regardless of firms' financing decisions or changes in retained earnings. However, net sales of stock cannot be changed by retained earnings or changes in stock price, unless firms decide to engage in a financial contract with external shareholders and sell shares.
Table 6 summarizes the size and the frequency of equity issuance in the sample. Approximately 75% of the observations are associated with positive gross stock sales, which suggests that equity issuance is common among listed firms in the U.S. In terms of size, approximately 40% of the observations are associated with gross stock sales greater than 1% of lagged total assets. It is also notable that a non-negligible fraction of observations are associated with sizable equity issuance relative to the existing size of the firm (greater than 3% of total assets). Net stock sales show a similar pattern. Approximately 50% of observations report positive net stock sales and approximately 20% of the observations are associated with net equity issuance that exceeds 3% of lagged total assets. Figure 11 shows the distribution of net and gross stock sales.

The vector $X_{i,t}$ of firm-level control variables includes cash flow, Tobin's Q, sales growth, asset growth,[18] and firm size as measured by the log of beginning-of-period total assets (or lagged total assets). These variables are chosen mostly following Korajczyk and Levy (2003), Fama and French (2005), Covas and Den Haan (2011), and Erel et al. (2012), all of

[18]Cash flow, sales growth, and asset growth are all normalized by lagged total assets. See Appendix for a precise definition of variables.

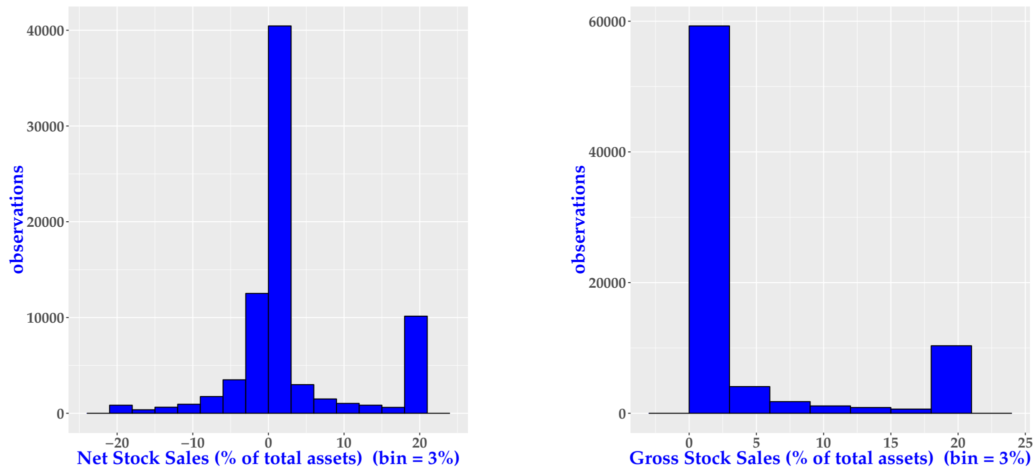
Table 6: Frequency of Equity Financing

Interval	Gross Stock Sales		Net Stock Sales	
	Frequency	Percentage	Frequency	Percentage
less than 0%	-	-	20,563	26.31%
0%	18,659	23.88%	13,592	17.39%
0∼1%	30,060	38.46%	20,180	25.82%
1∼3%	10,563	13.52%	6,681	8.55%
greater than 3%	18,867	24.14%	17,133	21.92%
Total observations	78,149	100%	78,149	100%

Notes: The table shows the number of observations and relative frequency of positive gross and net stock sales. Gross and net stock sales are defined as in Table 5; Net stock sales are measured as sales of common and preferred stock (SSTK) minus purchases of common and preferred stock (PRSTKC) normalized by beginning-of-year total assets, or lagged total assets (AT). Gross stock sales are measured as sales of common and preferred stock (SSTK) normalized by beginning-of-year total assets.
Source: Compustat

which report firm-level variables that are closely related to firms' financing decisions. Higher cash flows imply that firms have more internal funds to finance production and investment projects. The pecking order theory predicts that firms prefer internal sources of funds to external financing, since external financing is more expensive due to information asymmetry problems that affect financial contracts. Hence, firms with higher cash flow are expected to have lower net stock sales. Tobin's Q measures firms' growth opportunities. If firms have higher growth opportunities, they are more likely to raise external funds, in addition to using internal funds. This implies that firms with higher Tobin's Q are likely to have higher net stock sales. Sales growth is potentially related to external financing in two opposite ways. Holding firms' profitability constant, higher sales growth possibly implies that more internal funds are available, in which case firms are less likely to issue equity to finance production and investment projects. However, it is also possible that firms experience higher sales growth when they are expanding their business rapidly. In this case, firms may need to issue more stock to meet financing needs that cannot be covered solely with internal funds. Finally, higher asset growth is likely to be associated with positive net stock sales, since faster growing firms are less likely to be able to meet their financing needs solely with

Figure 11: Distribution of Net and Gross Stock Sales



Notes: The left panel shows the distribution of net stock sales. The right panel shows the distribution of gross stock sales. Gross and net stock sales are defined as in Table 5; Net stock sales are measured as sales of common and preferred stock (SSTK) minus purchases of common and preferred stock (PRSTKC) normalized by beginning-of-year total assets, or lagged total assets (AT). Gross stock sales are measured as sales of common and preferred stock (SSTK) normalized by beginning-of-year total assets. Capital letters in parentheses represent Compustat variable mnemonics. The size of bin is 3%. For both panels, the largest bin includes all observations greater than 20%. The smallest bin includes all observations less than -20%. See Table 6 for more details of the distribution.
Source: Compustat

internal funds.

Firm size is included since firms in different size classes exhibit different financing patterns over the business cycle. Covas and Den Haan (2011) document that smaller firms tend to exhibit stronger procyclicality for both debt and equity financing compared to larger firms, while in contrast the largest firms (top 1% of Compustat firms) raise equity countercyclically.

$Equity\ finance_{i,t}$ is winsorized at 300% and -300%.[19] All other firm-level variables are winsorized at the top and bottom 0.5 percentiles.

Table 7 summarizes the estimation results of equation (16). I use a least square dummy variable (LSDV) estimator to estimate the panel regression model. To show that the sample is consistent with the previous empirical literature on the cyclicality of equity financing, I first estimate the regression model without $Uncertainty$. Spec 1 shows that equity financing is

[19]The top and bottom 0.5 percentiles of $Equity\ finance_{i,t}$ are 957% and -25% respectively. Since the distribution of $Equity\ finance_{i,t}$ is substantially skewed, winsorizing at the top and bottom 0.5 percentiles seems insufficient. However, the baseline results do not change in case of winsorizing at the top and bottom 0.5 percentiles.

positively correlated with real GDP growth both contemporaneously and in lags, consistent with previous empirical literature.

Spec 2 - 3 show that the level of uncertainty is negatively correlated with firms' equity financing decisions in lags. Spec 4, which controls for the largest set of firm characteristics, suggests that the level of uncertainty is negatively correlated with equity financing, both contemporaneously and in lags. Results are statistically significant at 1% for all specifications. The economic significance of uncertainty is also non-negligible. A one standard deviation marginal increase in uncertainty, which is 0.018, results in a roughly 0.7 percentage point decrease in net sales of stock both simultaneously and in lags.[20] In other words, an average firm with total assets of $3,200 million constant U.S. dollars reduces net stock sales by $22 million constant U.S. dollars when the measure of uncertainty increases by one standard deviation.

All specifications suggest that equity financing is negatively correlated with cash flow and positively related to Tobin's Q, consistent with the prediction of corporate finance theories discussed above. Sales growth is negatively correlated with equity financing (Spec 3 and Spec 4). The result is in line with the hypothesis that an increase in total sales, controlling for asset growth and cash flow, implies sufficient internal funds that firms are less inclined to rely on external financing. However, a negative correlation between sales growth and net sales of equity is not consistent with Erel et al. (2012), who report a strong positive relationship between sales growth and seasoned equity offerings. The opposite result is mostly due to the inclusion of asset growth. As Erel et al. (2012) do not control for asset growth, the effect of sales growth is confounded with the effect of asset growth. Indeed, the regression results show that asset growth is positively correlated with equity financing. This is consistent with corporate finance theories predicting that internal funds are not fully sufficient to meet the financing needs of fast growing firms, and thus these firms must tap an external source of funds such as debt or equity. Lastly, firm size is negatively correlated with equity financing, which suggests that smaller firms may be more likely to operate at a

[20]Note that net sales of equity are normalized by lagged total assets. So, the exact interpretation of the point estimate is that there is a 0.7 percentage point decrease in the "net equity sales-to-total assets ratio." in response to a 1 standard deviation increase in uncertainty.

Table 7: Aggregate uncertainty and net stock sales

	(1) Spec 1	(2) Spec2	(3) Spec 3	(4) Spec 4
$RGDP\, growth_t$	0.0077***	0.0078***	0.0061***	-0.00020
	(9.91)	(8.38)	(6.50)	(-0.21)
$RGDP\, growth_{t-1}$	0.0024***	0.0021*	0.0016	0.0013
	(3.18)	(1.80)	(1.44)	(1.17)
$Cashflow_{i,t}$	-0.20***	-0.20***	-0.12***	-0.12***
	(-14.80)	(-14.84)	(-8.57)	(-8.47)
$Tobin's\, Q_{i,t}$	0.035***	0.035***	0.015***	0.013***
	(42.07)	(42.09)	(15.71)	(13.61)
$Sales\, growth_{i,t}$			-0.015*	-0.031***
			(-1.93)	(-4.20)
$Total\, asset\, growth_{i,t}$			0.16***	0.15***
			(25.55)	(24.03)
$Firm\, size_{i,t}$				-0.092***
				(-25.01)
$Uncertainty_t$		-0.0097	0.093	-0.40***
		(-0.08)	(0.80)	(-3.46)
$Uncertainty_{t-1}$		-0.30***	-0.30***	-0.40***
		(-2.89)	(-3.06)	(-4.10)
Constant	-0.030***	0.048**	0.057**	0.74***
	(-8.86)	(2.03)	(2.54)	(20.51)
Adjusted R^2	0.417	0.417	0.490	0.507
Observations	78149	78149	78149	78149

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table summarizes firm fixed effects panel regression results using a least square dummy variable (LSDV) estimator. The dependent variable is $Equity\, finance_{i,t}$ for all specifications. All specifications include firm fixed effects. See Appendix for the detailed definition of firm-level control variables. Numbers in parentheses are t-statistics with adjusted standard errors clustered by firms.

smaller-than-optimal size, and thus need more funds to reach optimal size.

6.3 Robustness

In this section, I show that the empirical results in the previous section are robust to using alternative measures of uncertainty.

As a first exercise, I construct a measure of uncertainty based on firm-level stock returns ($Uncertainty_t^{Stock}$) following Bloom et al. (2012).[21] In particular, I calculate the standard deviation of monthly stock returns across months and firms within a year using CRSP.[22] As Bloom et al. (2012) point out, a stock return-based uncertainty measure has one notable advantage over TFP-based uncertainty measures. The residuals from estimating equation (15) are productivity shocks in the sense that they are not forecasted by the regression equation, but this does not necessarily imply that the residuals are not forecasted by firms. In contrast, $Uncertainty_t^{Stock}$ is immune to such concerns. The estimation result is reported in the 1st column of Table 8. Stock return-based uncertainty is negatively correlated with equity finance. A one standard deviation increase in $Uncertainty_t^{Stock}$ (0.027) results in an 0.7 percentage point decrease in net stock sales in lags. The statistical and economic significance of $Uncertainty_t^{Stock}$ is similar to other uncertainty measures. Results using a stock return-based uncertainty measure constructed at an industry level ($Uncertainty_t^{Stock-IND}$), reported in the 2nd column, show slightly stronger economic significance. To be more concrete, a one standard deviation in $Uncertainty_t^{Stock-IND}$ (0.028) results in a 1 percentage point decrease in net sales of stock in lags.

As a second exercise, I use the CBOE S&P 100 Volatility Index (VIX_t), which is a widely used proxy of aggregate uncertainty.[23] The result is reported in the third column, and it strongly supports the hypothesis that uncertainty adversely affects equity financing. A one standard deviation increase in the VIX_t (7.1) results in a 1.4 percentage point decrease in net sales of stock contemporaneously, and 0.6 percentage point in lags.

[21] The correlation between $Uncertainty_t$ and $Uncertainty_t^{Stock}$ is 0.79.

[22] Observations are excluded from the calculation if fewer than 6 months of observations are available per year per firm.

[23] The correlation between $Uncertainty_t$ andVIX_t is 0.55.

Table 8: Alternative Uncertainty Measures

	(1)	(2)	(3)
	$Equity\,finance_t$	$Equity\,finance_t$	$Equity\,finance_t$
$RGDP\,growth_t$	0.00048	0.0018*	-0.0025***
	(0.48)	(1.79)	(-2.77)
$RGDP\,growth_{t-1}$	-0.0010	-0.0025***	-0.0019**
	(-1.08)	(-2.75)	(-2.22)
$Cashflow_{i,t}$	-0.12***	-0.12***	-0.12***
	(-8.46)	(-8.42)	(-8.60)
$Tobin's\,Q_{i,t}$	0.013***	0.013***	0.013***
	(13.63)	(13.66)	(13.65)
$Sales\,growth_{i,t}$	-0.031***	-0.031***	-0.031***
	(-4.23)	(-4.21)	(-4.26)
$Total\,asset\,growth_{i,t}$	0.15***	0.15***	0.15***
	(24.03)	(24.02)	(24.07)
$Firm\,size_{i,t}$	-0.091***	-0.091***	-0.089***
	(-25.02)	(-24.94)	(-24.88)
$Uncertainty_t^{Stock}$	-0.099		
	(-1.23)		
$Uncertainty_{t-1}^{Stock}$	-0.25***		
	(-3.51)		
$Uncertainty_{j,t}^{Stock-IND}$		0.12	
		(1.58)	
$Uncertainty_{j,t-1}^{Stock-IND}$		-0.35***	
		(-5.21)	
VIX_t			-0.0020***
			(-8.86)
VIX_{t-1}			-0.00081***
			(-3.41)
Adjusted R^2	0.507	0.506	0.508
Observations	78149	78149	78149

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table summarizes firm fixed effects panel regression results using a least square dummy variable (LSDV) estimator replacing aggregate uncertainty, $Uncertainty_t$, with alternative measures of uncertainty. All specifications include firm fixed effects. See Appendix for the detailed definition of firm-level control variables. Numbers in parentheses are t-statistics using adjusted standard errors clustered by firms.

As a third exercise, I investigate if firm size is related to the degree to which uncertainty affects firms' equity financing decisions. This exercise is motivated by Covas and Den Haan (2011), who document that equity financing is significantly procyclical, and that the procyclicality increases as firm size decreases. I introduce interaction terms between uncertainty and firm size, and between real GDP growth and firm size. Results are reported in the first column of Table 9. The coefficients of the interaction terms between uncertainty and firm size are positive and significant, which implies that smaller firms' equity financing decisions are more severely and adversely affected by an increase in uncertainty.[24]

As a fourth exercise, I investigate how debt financing is affected by uncertainty. I re-estimate equation (16) replacing the dependent variable with $Debt\,finance_{i,t}$, which is changes in total debt normalized by lagged total assets. The result is reported in the second column of Table 9. The results suggest that debt financing also decreases as uncertainty increases, and the sensitivity increases as firm size decreases. Replacing the dependent variable with $leverage_{i,t}$, which is the ratio between total debt and beginning-of-period total assets, shows a similar result (the third column of Table 9). As uncertainty increases, leverage decreases.

As a fifth exercise, I investigate how industry-level uncertainty affects firms' equity financing decisions. This exercise addresses a concern that different industries potentially have a different degree of idiosyncratic productivity dispersion in a given year. The measure of industry-level uncertainty is constructed similarly as in the aggregate uncertainty measure, $Uncertainty_t$, discussed in the previous section. The only difference is that I take the standard deviation of OLS residuals $\hat{u}_{i,j,t}$ of equation (15) at the two-digit NAICS level within a year. Hence, industry-level uncertainty, $Uncertainty^{IND}_{j,t}$, varies not only by t but also by industry j. The fourth column of Table 9 presents estimation results of equation (16) replacing $Uncertainty_t$ with $Uncertainty^{IND}_{j,t}$. Implications are identical to the baseline results reported in Table 7. Industry level uncertainty is negatively correlated with equity financing both contemporaneously and in lags, and the results are statistically significant at 1%. In all specifications, I control for the business cycle using contemporaneous and lagged real

[24]The lagged interaction term between real GDP growth and firm size is negative and significant, which implies that smaller firms' equity financing decisions are more sensitive to the business cycle. This result is consistent with Covas and Den Haan (2011).

Table 9: Robustness

	(1) $Equity\,finance_{i,t}$	(2) $Debt\,finance_{i,t}$	(3) $Leverage_{i,t}$	(4) $Equity\,finance_{i,t}$	(5) $Equity\,finance_{i,t}$
$RGDP\,growth_t$	0.0029	-0.020***	-0.014***	0.011***	
	(0.93)	(-7.51)	(-7.35)	(4.05)	
$RGDP\,growth_{t-1}$	0.028***	0.019***	0.017***	0.0035	
	(6.72)	(5.19)	(6.96)	(1.25)	
$RGDP\,growth_t \times Firm\,size_{i,t}$	-0.00056	0.0042***	0.0032***	-0.0018***	-0.0019***
	(-1.28)	(10.67)	(11.43)	(-4.60)	(-4.97)
$RGDP\,growth_{t-1} \times Firm\,size_{i,t}$	-0.0047***	-0.0032***	-0.0027***	-0.00070*	-0.00053
	(-7.95)	(-6.13)	(-7.46)	(-1.79)	(-1.36)
$Cashflow_{i,t}$	-0.14***	0.013	-0.046***	-0.13***	-0.13***
	(-9.56)	(1.06)	(-5.69)	(-8.88)	(-9.15)
$Tobin's\,Q_{i,t}$	0.013***	-0.013***	-0.0089***	0.013***	0.013***
	(13.85)	(-13.83)	(-13.24)	(13.69)	(14.19)
$Sales\,growth_{i,t}$	-0.054***	0.052***	0.042***	-0.042***	-0.044***
	(-7.21)	(6.65)	(8.11)	(-5.69)	(-6.01)
$Total\,asset\,growth_{i,t}$	0.083***	0.084***	0.074***	0.12***	0.12***
	(10.81)	(9.87)	(13.06)	(17.06)	(17.10)
$Firm\,size_{i,t}$	-0.29***	-0.13***	-0.11***	-0.19***	-0.18***
	(-21.21)	(-11.04)	(-16.03)	(-21.00)	(-19.30)
$Uncertainty_t$	-3.84***	-2.82***	-2.56***		
	(-9.12)	(-8.12)	(-11.03)		
$Uncertainty_{t-1}$	-2.30***	-0.27	-0.018		
	(-6.81)	(-0.96)	(-0.09)		
$Uncertainty_t \times Firm\,size_{i,t}$	0.63***	0.61***	0.55***		
	(10.17)	(11.50)	(15.57)		
$Uncertainty_{t-1} \times Firm\,size_{i,t}$	0.32***	0.014	0.051*		
	(6.78)	(0.35)	(1.85)		
$Uncertainty^{IND}_{j,t}$				-1.03***	-1.02***
				(-7.64)	(-7.65)
$Uncertainty^{IND}_{j,t-1}$				-1.20***	-1.13***
				(-9.45)	(-8.94)
$Uncertainty^{IND}_{t} \times Firm\,size_{i,t}$				0.19***	0.19***
				(9.57)	(9.71)
$Uncertainty^{IND}_{t-1} \times Firm\,size_{i,t}$				0.20***	0.20***
				(10.80)	(10.64)
Firm fixed effect	Yes	Yes	Yes	Yes	Yes
Time fixed effect	No	No	No	No	Yes
Adjusted R^2	0.523	0.191	0.286	0.514	0.520
Observations	78149	78149	78149	78149	78149

t statistics in parentheses

$^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Notes: The table summarizes firm fixed effects panel regression results using a least square dummy variable (LSDV) estimator with interaction terms between uncertainty and firm size, and real GDP growth and firm size. The dependent variable of the second and the third column is $Debt\,finance_{i,t}$ and $Leverage_{i,t}$ respectively. See Appendix for the detailed definition of firm-level control variables. Numbers in parentheses are t-statistics using adjusted standard errors clustered by firms.

GDP growth rates. Using industry-level uncertainty allows me to control for the business cycle in an alternative way, which is replacing $RGDP\,growth_t$ and $RGDP\,growth_{t-1}$ with a time fixed effect. This address a potential concern that uncertainty measures are falsely picking up business cycles instead of uncertainty. As the last column of Table 9 reports, the inclusion of the time-fixed effect yields an identical conclusion; equity financing is negatively correlated with uncertainty.

7 Concluding Remarks

In this research, I study how uncertainty shocks affect firms' financing decisions, in particular equity financing, and how equity finance affects the macroeconomic impact of uncertainty shocks. I build a DSGE model with endogenous debt and equity contracts to investigate macroeconomic consequences of uncertainty shocks working through financial frictions. In my model, firms reduce debt financing in response to increased uncertainty, as debt becomes more expensive due to the increased default probability. This is consistent with the predictions of standard financial accelerator models. The model also predicts a decrease in equity financing in response to higher uncertainty, consistently with the data.

Introducing an endogenous equity contract into a DSGE model is a unique feature of the model, and this feature is important since it affects macroeconomic dynamics in response to uncertainty shocks. Incorporating equity financing generates additional amplification of uncertainty shocks since firms reduce equity financing whenever uncertainty is high. Through this channel, uncertainty shocks are amplified relative to a model with only debt contracts.

By incorporating uncertainty shocks and their amplification through equity financing frictions, my model is also able to explain procyclical debt and equity financing along with countercylical external financing costs, a combination which existing models fail to explain.

I also provide empirical evidence on the relationship between uncertainty and equity financing using firm-level data from Compustat. I show that firms reduce both debt and equity financing in response to higher uncertainty.

I mention two directions for future research. First, it is widely assumed that different classes of firms have varying degrees of financial constraints. Existing research suggests that smaller firms are more sensitive to uncertainty shocks, and my empirical results show that smaller firms' debt and equity finance decisions are more sensitive to uncertainty. However, the model in this paper is silent regarding potential heterogeneity in the effect of uncertainty shocks on equity financing decisions. Future research should model heterogeneous responses of firms' financing decisions to uncertainty shocks.

Secondly, this paper studies mostly uncertainty shocks. Since the financial contract that I propose in this paper can easily be embedded in other representative agent DSGE models, the simple model in this paper can be extended almost immediately to medium/large scale models. In this regard it will also be interesting to see how the equity finance channel affects the economy's responses to other aggregate shocks, such as monetary policy shocks.

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Appendix

TFP Estimation

In this section of the appendix, I provide details of how variables used in the TFP estimation are constructed. I heavily follow Covas and Den Haan (2011) and Imrohoroglu and Tüzel (2014) in cleaning data and defining variables. All balance sheet variables are from Compustat. Capital letters in parentheses represent Compustat mnemonics. Financial and utility firms (SIC 4900-4949 and 6000-6999, NAICS 22 and 52-53) are deleted from the sample, which is a standard procedure in the literature. I further drop postal service, courier and messengers, and the warehousing and storage industry (NAICS 49), since less than 250 observations are available for the entire sample period. Firms belonging to an unclassified industry (NAICS 99) are dropped from the sample. Firm-year observations with non-positive total assets (AT), total sales (SALE), operating income before depreciation and amortization (OIBDP), number of employees(EMP), gross property, plant, and equipment (PPEGT), or net property, plant, and equipment (PPENT) are dropped from the sample. These variables are necessary inputs to construct variables used in the TFP regression. To ensure data reliability, firm-year observations violating the accounting identity (total assets equals total liabilites plus stockholders' equity) by more than 10% are dropped from the sample. Observations with a fiscal year ending between April and August are dropped from the sample in order to minimize calendar and fiscal year mismatch. Value-added ($VA_{i,j,t}$)is defined as total sales (SALE) net of materials. Materials are defined as total expenses minus labor expenses. Total expenses are measured as total sales (SALE) minus operating income before depreciation and amortization (OIBDP), and labor expenses are defined as the number of employees (EMP) times the industry-level average annual wage. I mostly use 2-digit NAICS-level industry wages. I use 3- or 4-digit industry level wages if available. The source of the average annual wage data is the Bureau of Labor Statistics [link]. I mostly use a 2-digit NAICS level value-added price deflator to calculate real values of value-added. However I use 3- or 4-digit industry level if available. The source of the value-added price deflators is the Bureau of Economic Analysis. The number of employees, $L_{i,j,t}$ is directly available from Compustat (EMP). I define the stock of capital $K_{i,j,t}$ as lagged net property, plant, and equipment (PPENT). However, measuring the real capital stock is not straightforward since investments are made at different points of time. To address this issue, I calculate an average age of capital, and deflate the capital stock accordingly using an aggregate non-residential fixed investment deflator from the Bureau of Economic Analysis, which can easily be downloaded from Federal Reserve Economic Data.[25] For example, if a firm's average age of capital in 2000 is 3 years, I deflate net property, plant, and equipment (PPENT) using its deflator in 1997. Average age of capital is defined as a 3-year moving average of accumulated depreciation (DPACT) divided by current depreciation (DP). Intermediate inputs are defined as materials, which are defined above. I mostly use 2-digit NAICS level industry intermediate input price deflators. However I use 3- or 4-digit industry level data if available. The source of data is the Bureau of Labor Statistics.

The TFP estimation yields a total of 68,964 firm-year TFP observations for 8,500 unique firms (which is approximately 85% of the sample included in the main regression.) Note that this is *not* identical to the number of firm-year observations and unique firms which are used to estimate equation (16). There are fewer observations available for the variables required for the TFP regression than for the main regression. I do not necessarily require observations included in the main regression to be included in the TFP regression. This procedure maximizes the number of observations for the main regression.

Variable Definition

On variable definitions and firms included in the main regression sample, I mostly follow Covas and Den Haan (2011). Capital letters in parenthesis represent Compustat mnemonics. Financial and utility firms (SIC 4900-4949 and 6000-6999, NAICS 22 and 52-53) are deleted from the sample, which is a standard procedure in the literature. I further drop postal service, courier and messengers, and warehousing and storage (NAICS 49), since

[25] Although Bureau of Labor Statics provides [link] an investment good deflator at 2- and 3-digit NAICS level, the series starts only in 1990. Since the stock of capital is built by a sequence of investment over time, it is necessary to know the price deflator of investment goods purchased prior to 1990, unless all investments are made after 1990. For the reason, using an industry level deflator of investment causes a substantial loss of observations.

less than 250 observations are available for the entire sample period. Firms belonging to an unclassified industry (NAICS 99) are dropped from the sample. Firm-year observations with negative total liabilities (LT), long-term debt (DLTT), debt in current liabilities (DLC), stock price (PRCC_F), liquidating value of preferred stock (PSTKL), dividends to preferred stock (DVP), sales of common and preferred stocks (SSTK), and purchases of common and preferred stocks (PRSTKC) are dropped from the sample. Firm-year observations with non-positive stockholders' equity (SEQ) are dropped from the sample. If firms are involved in major mergers (Compustat footnote code AB), they are excluded from the sample. GM, GE, Ford, and Chrysler are dropped from the sample since they are substantially affected by accounting changes in 1988. If a firm-year observation violates the accounting identity (total assets (AT) equals total liabilites (LT) plus stockholders' equity(SEQ)) by more than 10%, it is dropped from the sample. Firm-year observations with stockholders' equity (SEQ)-to-total asset (AT) ratio below 0.01 or total debt-to-total asset (AT) ratio greater than 1 are dropped from the sample. Total debt is defined as the sum of debt in current liabilities (DLC) and long-term debt (DLTT). This restriction is to exclude firms which are virtually bankrupt, or in substantial financial distress. Lastly, observations with a fiscal year ending in between April and August are dropped from the sample in order to minimize calendar and fiscal year mismatch. The definitions of firm level balance sheet variables are as follows (Items in double quotation marks refer to variable names in the Compustat manual. All variables are deflated by the U.S. CPI):

- $Equity\,finance_t$: "**Sale of Common and Preferred Stock (SSTK)**" minus "**Purchase of Common and Preferred Stock (PRSTKC)**" divided by lagged "**Assets - Total (AT)**"
- $Debt\,finance_{i,t}$: Changes in total debt divided by lagged "**Assets - Total (AT)**" where total debt is "**Debt in Current Liabilities - Total (DLC)**" plus "**Long-Term Debt - Total (DLTT)**"
- $Leverage_{i,t}$: Total debt divided by lagged "**Assets - Total (AT)**" where total debt is "**Debt in Current Liabilities - Total (DLC)**" plus "**Long-Term Debt - Total (DLTT)**"
- $Cashflow_{i,t}$: "**Income Before Extraordinary Items (IB)**" minus "**Depreciation and**

Amortization (DP)" divided by lagged "**Assets - Total (AT)**"

- $Tobin's\, Q_{i,t}$: "**Common Shares Outstanding (CSHO)**" times "**Price Close - Annual - Fiscal (PRCC_F)**" plus "**Preferred Stock - Liquidating Value (PSTKL)**" plus "**Dividends - Preferred/Preference(DVP)**" plus "**Liabilities - Total (LT)**" divided by lagged "**Assets - Total (AT)**"
- $Sales\, growth\, rate_{i,t}$: "**Sales/Turnover (Net) (SALE)**" minus lagged "**Sales/Turnover (Net)**" divided by lagged "**Assets - Total (AT)**"
- $Asset\, growth\, rate_{i,t}$: "**Assets - Total (AT)**" minus lagged "**Assets - Total (AT)**" divided by lagged "**Assets - Total (AT)**"
- $firm\, size_{i,t}$: log of lagged "**Assets - Total (AT)**"